$\boxed{\text{LAST TIME:}}$

Wednesday 9/21/2016

↳ Limits [Main Trick: Polar Coordinates]

↳ Review

$\boxed{\text{TODAY:}}$

↳ Review & Problem Session

Example Problem: ★GOOD PRACTICE PROBLEM★

let $\vec{v}$ & $\vec{w}$ be vectors.

Show that $\vec{w} - \text{proj}_{\vec{v}}(\vec{w})$ and $\vec{v}$ are orthogonal.

Soln: We need to show that the dot product is zero.

$$\left(\vec{w} - \text{proj}_{\vec{v}}(\vec{w})\right)\cdot \vec{v} = \left(\vec{w} - \left(\vec{w}\cdot \frac{\vec{v}}{|\vec{v}|}\right)\frac{\vec{v}}{|\vec{v}|}\right)\cdot \vec{v}$$

$$= \vec{w}\cdot\vec{v} - \left(\left(\vec{w}\cdot\frac{\vec{v}}{|\vec{v}|}\right)\frac{\vec{v}}{|\vec{v}|}\right)\cdot\vec{v}$$

$$= \vec{w}\cdot\vec{v} - \left(\vec{w}\cdot\frac{\vec{v}}{|\vec{v}|}\right)\left(\frac{\vec{v}}{|\vec{v}|}\cdot\vec{v}\right)$$

$$= \vec{w}\cdot\vec{v} - \left(\frac{\vec{w}\cdot\vec{v}}{|\vec{v}|}\right)\left(\frac{|\vec{v}|^2}{|\vec{v}|}\right)$$

$$= \vec{w}\cdot\vec{v} - \vec{w}\cdot\vec{v}\,\cancel{\frac{|\vec{v}|^2}{|\vec{v}|^2}}^{\,1}$$

$$= 0 \,//$$

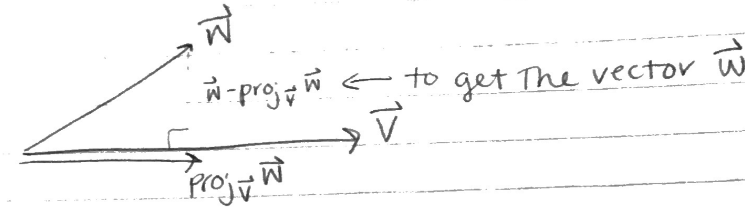
Other formula solution...

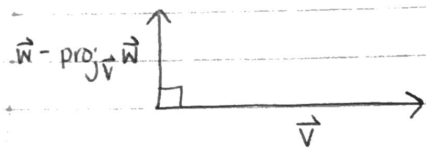
$$\left(\vec{w} - \text{proj}_{\vec{v}}(\vec{w})\right)\cdot\vec{v} = \vec{w} - \overbrace{\left(\frac{\vec{v}\cdot\vec{w}}{|\vec{v}|^2}\right)}^{\text{constant}}\cdot\vec{v}$$

$$= \vec{w}\cdot\vec{v} - \frac{\vec{v}\cdot\vec{w}}{\cancel{|\vec{v}|^2}}\cancel{(\vec{v}\cdot\vec{v})}$$

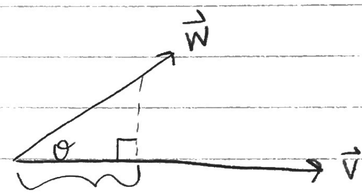
$$= \vec{w}\cdot\vec{v} - \vec{v}\cdot\vec{w} = 0 \,//$$

Idea of projections:





$$proj_{\vec{v}}(\vec{w}) = \underbrace{\left(\vec{w} \cdot \frac{\vec{v}}{|\vec{v}|}\right)}_{\text{amt of } \vec{w} \text{ in direction of } \vec{v}.} \frac{\vec{v}}{|\vec{v}|}$$



$|\vec{w}| \cos(\theta)$ ← scalar projection

$$\vec{w} \cdot \frac{\vec{v}}{|\vec{v}|} = |\vec{w}| \cancelto{1}{\left|\frac{\vec{v}}{|\vec{v}|}\right|} \cos\theta$$

$$= |\vec{w}| \cos\theta$$

$$\left(\begin{array}{c}\text{magnitude of vector}\\ \text{proj (of } \vec{w} \text{ on } \vec{v})\end{array}\right) = |\vec{w}| \cos(\theta)$$

$$\left(\begin{array}{c}\text{direction of vector}\\ \text{proj (of } \vec{w} \text{ on } \vec{v})\end{array}\right) = \left(\begin{array}{c}\text{direction}\\ \text{of } \vec{v}\end{array}\right) = \frac{\vec{v}}{|\vec{v}|}$$

Example of Vector Projection:

Find the vector projection of $\vec{a}$ onto $\vec{b}$ where

$\vec{a} = (5,6,7)$

$\vec{b} = (1,2,3)$

Soln: $\text{proj}_{\vec{b}}\vec{a} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

$$\vec{a}\cdot\vec{b} = (5,6,7)\cdot(1,2,3)$$
$$= 5+12+21 = 38$$

$$|\vec{b}| = \sqrt{1+4+9} = \sqrt{14}$$
$$|\vec{b}|^2 = 14$$

$$\text{proj}_{\vec{b}}\vec{a} = \left(\frac{38}{14}\right)\vec{b}$$
$$= \frac{38}{14}(1,2,3)$$
$$= \left(\frac{38}{14}, \frac{74}{14}, \frac{114}{14}\right) = \boxed{\left(\frac{19}{7}, \frac{38}{7}, \frac{57}{7}\right)}$$

Example of completing the square:

$x^2+2ax + \text{JUNK}$
$= (x-a)^2 - a^2 + \text{JUNK}$

Ex1: $2x^2-16x+10$

$2(x^2-8x+5)$

TRICK: $(x+a)^2 = x^2+2a+a^2$

$(x-4)^2 = x^2-8x+16$

so you have to -16

$2((x-4)^2-16+5)$

$\boxed{2((x-4)^2-11)}$

Ex2: $x^2+12x+4$

$(x+6)^2 = x^2+12x+36$

$(x+6)^2-36+4$

$\boxed{(x+6)^2-32}$

Ex3: Put the eqn into standard form (using methods to complete the square)

$$z + y^2 + 12y + 4 + 2x^2 - 16x + 10 = 0$$

soln: $y^2 + 12y = (y+6)^2 - 36$

$2x^2 - 16x = 2(x^2 - 8x) = 2[(x-4)^2 - 16]$

so... $z + y^2 + 12y + 2x^2 - 16x + 14$

$= z + (y+6)^2 - 36 + 2[(x^2-8x) - 16] + 14$

$= z + (y+6)^2 + 2(x-4)^2 - 36 + 14$

$= z + (y+6)^2 + 2(x-4)^2 = 54$ // ok

$$\boxed{z = -(y+6)^2 - 2(x-4)^2 + 54}$$

↳ traces are ellipses

↳ surface is an elliptic paraboloid w/ z-axis of symmetry

Know it looks like a bell b/c

$z \approx -r^2 + C$