LAST TIME: (before midterm #1) Monday 9/26/2016

↳ limits * key point → change to polar coordinates *

↳ partial derivatives

TODAY: Gradients & Tangent Planes to Surfaces

Gradient → "take the value of all partial derivatives and stick them in a vector"

Given a function $g(x,y,z)$, the gradient of g is

$$\boxed{\nabla g = \frac{\partial g}{\partial x}\hat{i} + \frac{\partial g}{\partial y}\hat{j} + \frac{\partial g}{\partial z}\hat{k}}$$

"nabla"

Examples:

1. If $h(x,y,z) = ze^{xy}$

$\nabla h = \nabla(h(x,y,z)) = (zye^{xy},\ zxe^{xy},\ e^{xy})$ //

Note:

$\frac{\partial}{\partial x}[ze^{xy}] = zye^{xy}$

$\frac{\partial}{\partial y}[ze^{xy}] = zxe^{xy}$

$\frac{\partial}{\partial z}[ze^{xy}] = e^{xy}$

Also, if $h(x,y,z) = h(1,2,3)$

$\nabla h = ((3)(2)e^2,\ (3)(1)e^2,\ e^2)$

$\nabla h = (6e^2, 3e^2, e^2)$ //

2. We can do 2D versions:

If $f(x,y) = x^2y + e^y$

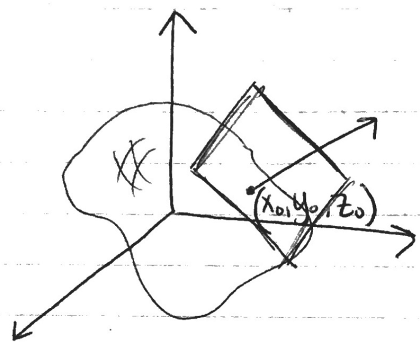
$\nabla f(x,y) = \frac{\partial}{\partial x}(x^2y + e^y)\hat{i} + \frac{\partial}{\partial y}(x^2y + e^y)\hat{j}$

$= 2xy\hat{i} + (x^2 + e^y)\hat{j}$ //

Tangent Planes

Fundamental question to Calc III

<u>Question:</u> Given a surface $g(x, y, z) = 0$ and a point (x_0, y_0, z_0) on that surface, find the plane tangent to that surface at the point (x_0, y_0, z_0).



we need to figure out what the normal vector at a point is.

SPOILER: The normal vector to the surface at the point (x_0, y_0, z_0) is $\nabla g(x_0, y_0, z_0)$ (the gradient)

This spoiler gives us the eqn for the tangent plane at a point.

$$\boxed{\nabla g(x_0, y_0, z_0) \cdot (x - x_0, y - y_0, z - z_0) = 0}$$ KEY FORMULA!

~~example: find the eqn,~~

$x^2+y^2+z^2=1$

example: Find an equation for the plane tangent to the unit sphere at the point $(x_0, y_0, z_0) = 1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}$.

In our application:

$$x^2+y^2+z^2=1 \iff x^2+y^2+z^2-1=0$$

ie: $g(x,y,z) = x^2+y^2+z^2-1$

compute the gradient:

$$\nabla g = (2x, 2y, 2z)$$
$$\nabla g\left(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}\right) = \left(2/\sqrt{3}, 2/\sqrt{3}, 2/\sqrt{3}\right)$$

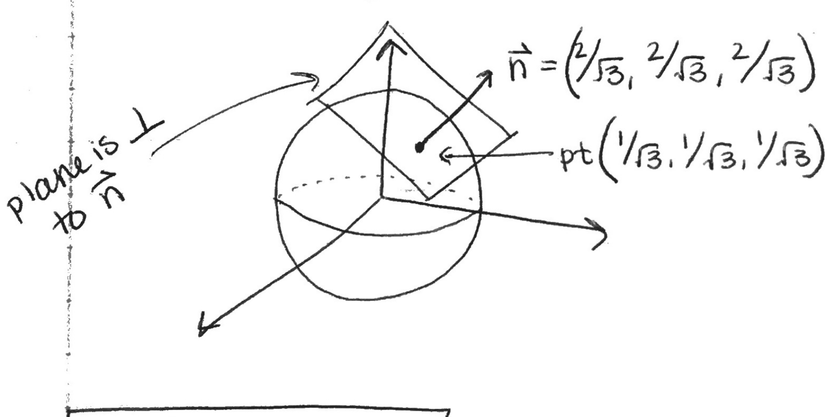
We now use the formula

$$\nabla g(x_0, y_0, z_0) \cdot (x-x_0, y-y_0, z-z_0) = 0$$

$$\left(2/\sqrt{3}, 2/\sqrt{3}, 2/\sqrt{3}\right) \cdot \left(x-1/\sqrt{3}, y-1/\sqrt{3}, z-1/\sqrt{3}\right) = 0 \quad //$$ This is an ok answer

$$\iff 2/\sqrt{3}\left(x-1/\sqrt{3}\right) + 2/\sqrt{3}\left(y-1/\sqrt{3}\right) + 2/3\left(z-1/\sqrt{3}\right) = 0$$
$$\iff x+y+z-3/\sqrt{3}=0$$
$$\iff x+y+z-\sqrt{3}=0 \quad //$$ completely simplified answer

Picture: $x^2+y^2+z^2=1$



$\boxed{\text{Btw (from midterm):}}$

$$x^2+y^2+z^2=1$$
$$z^2=1-x^2-y^2$$
$$z=\pm\sqrt{1-x^2-y^2}$$

the (+) part is the upper hemisphere.

example 2: Find the plane tangent to the graph of the function

$$f(x,y) = 1-x^2-y^2$$

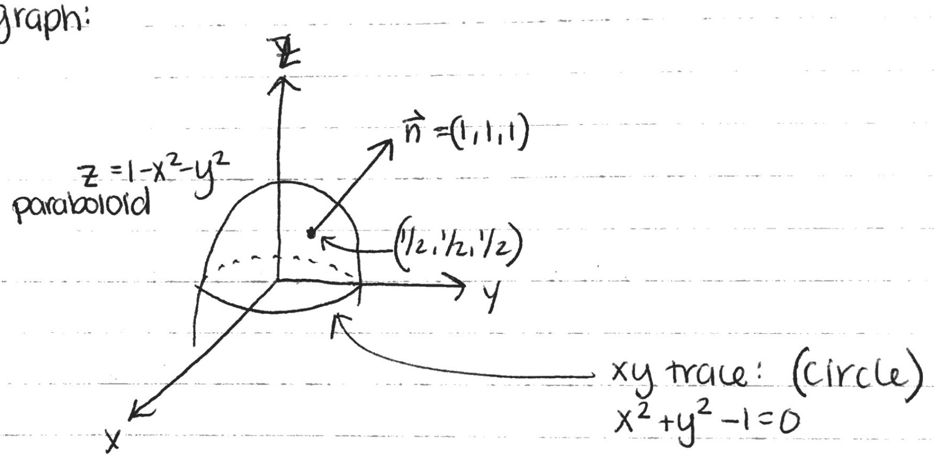
at the point $(1/2, 1/2, 1/2)$

Soln:

1. Check if the point is on the unit circle:

$$f(\tfrac{1}{2},\tfrac{1}{2}) = 1-(1/2)^2-(1/2)^2$$
$$= 1 - 1/4 - 1/4$$
$$= 1/2 \checkmark$$

2. graph:



3. $g(x,y,z) = z - f(x,y)$

gradient:

$$\nabla g = (2x, 2y, 1)$$
$$\nabla g(1/2, 1/2, 1/2) = (1,1,1)$$

4. The equation of the tangent plane is:

$$(1,1,1) \cdot (x-1/2,\ y-1/2,\ z-1/2) = 0$$
$$(x-1/2,\ y-1/2,\ z-1/2) = 0 \text{ // this form is ok}$$

$$\Longleftrightarrow \quad x+y+z - 3/2 = 0 \text{ //}$$