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Directional Derivatives:

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$\vec{r} = (x, y)$

Let $\vec{u}$ be a constant unit vector

$$f = f(\vec{r}) = f(x, y)$$

$$\boxed{D_{\vec{u}} f = \lim_{h \to 0} \frac{f(\vec{r} + h\vec{u}) - f(\vec{r})}{h}}$$

$D_{\vec{u}} f(\vec{r}) = \left(\text{The directional derivative of } f \text{ in the direction of } \vec{u}\right)$

<u>Idea:</u> Looking at the change in a certain direction.

$$D_{\vec{u}} F(\vec{r}) = \frac{d}{dt}\left[F(\vec{r} + t\vec{u})\right]$$

$$= \lim_{\Delta t \to 0} \frac{f(\Delta t + 0) - f(0)}{\Delta t}$$

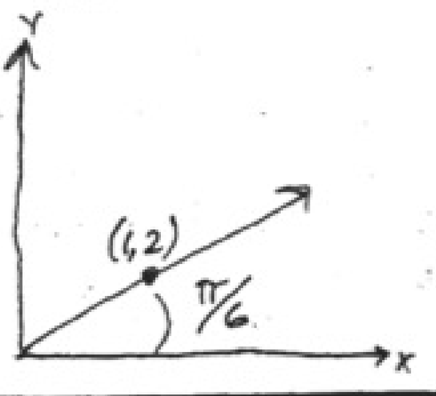
$$= \lim_{\Delta t \to 0} \frac{f(\vec{r} + (\Delta t + 0)\vec{u}) - f(\vec{r} + 0\vec{u})}{\Delta t}$$

$$D_{\vec{u}} f(\vec{r}) = \lim_{\Delta t \to 0} \frac{f(\vec{r} + \Delta t\,\vec{u}) - f(\vec{r})}{\Delta t}$$

$$\Longrightarrow D_{\vec{u}} f(\vec{r}) = \nabla f(\vec{r}) \cdot \vec{u}$$

- <u>Example problem:</u> Find the directional derivative of $F(x,y) = 3x^2 - 3xy - y^2$ at the point $(1,2)$ in the direction of $\theta = \pi/6$

- <u>Solution:</u>



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$\rightarrow \vec{u} = (\cos(\frac{\pi}{6}), \sin(\frac{\pi}{6}))$

$= (\frac{\sqrt{3}}{2}, \frac{1}{2}) \longrightarrow \nabla f = (6x - 3y, -3x - 2y)$

$\nabla f(1,2) = (6(1) - 3(2), -3(1) - 2(2))$

$\nabla f(1,2) = (0, 7)$

$D_{\vec{u}} f(1,2) = \nabla f(1,2) \cdot \vec{u} = (0,7) \cdot (\frac{\sqrt{3}}{2}, \frac{1}{2})$

$\boxed{= \frac{-7}{2}}$

• Max & Mins of Functions:

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Definition: A function f has a local max (or min) at the point (a, b) if $f(a,b) \geq f(x,y)$ for all (x,y) in a ball around (a,b).
($f(a,b) \leq (x,y)$ for local min)

- f has a global max or min if the inequality holds for all (x,y) in the domain of f

Theorem: If f has a local max or min at (a,b) and its first order derivatives (f_x & f_y) exist, then...

$$f_x(a,b) = f_y(a,b) = 0$$

note: f_x & f_y don't need to exist at a max or a min.

Second Derivative Test:

$$f_x(a,b) = f_y(a,b) = 0 \qquad D = \#$$

$$D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - (f_{xy}(a,b))^2$$

a.) if $D > 0$ and $f_{xx}(a,b) > 0$, then (a,b) is a local min.
b.) if $D > 0$ and $f_{xx}(a,b) < 0$, then (a,b) is a local max.
c.) if $D < 0$ then (a,b) is not a max or min, but a saddle point.