Math 121 — Homework 01

Instructions Remember to show all of your work to get credit. Please do this assignment on a separate sheet of paper. The assignment is due at the beginning of class on Tuesday. Remember to use your words.

1. (a) Show that $\vec{a} \times \vec{b}$ and $\vec{a}$ are orthogonal.

   (b) Prove the Cauchy-Schwarz inequality:

   $$|\vec{a} \cdot \vec{b}| \leq |\vec{a}||\vec{b}|.$$

   (Hint: use the angle formula for dot products and the fact that $|\cos(\theta)| \leq 1$.)

2. Consider the plane that passes through points P, Q, R in $\mathbf{R}^3$. Show that the distance from the plane containing P, Q and R to a point S is

   $$\left| \overrightarrow{PS} \cdot \frac{\overrightarrow{PQ} \times \overrightarrow{PR}}{|\overrightarrow{PQ} \times \overrightarrow{PR}|} \right|$$

   where $\vec{u} = \overrightarrow{PQ}$, $\vec{v} = \overrightarrow{PR}$, $\vec{w} = \overrightarrow{PS}$. (Hint: There are a couple ways to do this. For me, the simplest is to think about vector projections. You should always draw the picture when thinking about these things.)