Math 121 — Homework 02

Instructions Remember to show all of your work to get credit. Please do this assignment on a separate sheet of paper. The assignment is due at the beginning of class on Friday. Use you

1. Consider the hyperbolic paraboloid X defined by
$$z = y^2 - x^2.$$
   (a) For a point (a,b,c) lying on X show that the line
$$\begin{cases} x = a + t, \\ y = b + t, \\ z = c + 2(b-a)t \end{cases},$$
   lies on the surface of X.

   (b) Draw the hyperbolic paraboloid X and the line for the point $(1,1,0)$.

   (Surfaces where every point has a line through it completely contained in the surface is called "ruled". This exercise shows that this hyperbolic paraboloid is ruled.)

2. Prove the product rule for the cross product:
$$\frac{d}{dt}(\mathbf{u}(t) \times \mathbf{v}(t)) = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t).$$
   (Hint: Use the definition of $\mathbf{u}(t) \times \mathbf{v}(t)$ and the definition of the derivative on components. If it feels tedious you are doing it right.)

3. Prove that the derivative of angular momentum is torque:
$$\frac{d}{dt}[\vec{l}(t)] = \vec{\tau}(t).$$
   Here we recall that for a particle of mass m and position $\vec{r}(t)$ we have the following definitions:
$$(\text{angular momentum}) = \vec{l}(t) = \vec{r}(t) \times \vec{p}(t)$$
$$(\text{torque}) = \vec{\tau}(t) = \vec{r}(t) \times \vec{F}(t).$$
$$(\text{momentum}) = \vec{p}(t) = m\dot{\vec{r}}(t)$$
$$(\text{force}) = \vec{F}(t) = m\ddot{\vec{r}}(t)$$
   (Hint: The answer is one line.)

4. Suppose that $\mathbf{r}(t)$ is a vector valued function such that $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 0$. Show that $\mathbf{r}(t)$ lives on a circle centered at the origin. (Hint: Write $\mathbf{r}(t) = (x(t), y(t), z(t))$ and take the derivative of the function $|\mathbf{r}(t)|^2$ after expanding everything out.)