

Math 121 — Homework 04

Instructions Remember to show all of your work to get credit. Please do this assignment on a separate sheet of paper. Remember to use your words when explaining something.

1. For a function $u = u(x, t)$ the wave equation is given by

$$u_{tt} - c^2 u_{xx} = 0.$$

where c is a constant. If f and g are functions of a single variable show that $u(x, t) = f(x - ct) + g(x + ct)$, gives a solution of the wave equation.

2. The ideal gas law states that

$$PV = nRT,$$

where

- P is the pressure of the gas,
- V is the volume of the gas,
- n is the amount of substance of gas (in moles),
- R is the ideal, or universal, gas constant, equal to the product of the Boltzmann constant and the Avogadro constant,
- T is the absolute temperature of the gas.

- (a) Show that

$$\frac{\partial P}{\partial V} \frac{\partial V}{\partial T} \frac{\partial T}{\partial P} = -1.$$

- (b) Show that

$$T \frac{\partial P}{\partial T} \frac{\partial V}{\partial T} = nR.$$

3. If $u = e^{a_1 x_1 + a_2 x_2 + \dots + a_n x_n}$, where $a_1^2 + a_2^2 + \dots + a_n^2 = 1$ show that

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \dots + \frac{\partial^2 u}{\partial x_n^2} = u.$$