Dupuy — Math 121 — Homework 05

Instructions Remember to show all of your work to get credit. Please do this assignment on a separate sheet of paper. Remember to use your words.

1. Consider a function $w = f(x, y)$ where $x = r\cos(\theta)$ and $y = r\sin(\theta)$. Prove the following

   (a)
   $$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial r}\cos(\theta) - \frac{\partial w}{\partial \theta}\frac{\sin(\theta)}{r}$$
   $$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial r}\sin(\theta) + \frac{\partial w}{\partial \theta}\frac{\cos(\theta)}{r}$$

   (b)
   $$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2$$

2. A function $f(x, y)$ is said to be homogeneous of degree d if
   $$f(tx, ty) = t^d f(x, y).$$

   (a) Show that $f(x, y) = x^3 - 3xy^2 + y^3$ is homogeneous. What is its degree?

   (b) Show that if $g(x, y)$ is homogeneous of degree d with $d \geq 1$ then
   $$x\frac{\partial g}{\partial x} + y\frac{\partial g}{\partial y} = dg(x, y).$$
   (Hint: Compute $\frac{\partial}{\partial t}[g(tx, ty)]$ in two different ways.)

3. Two surfaces are said to be orthogonal at a point P if the normal lines at those points are perpendicular. Show that the surfaces $z^2 = x^2 + y^2$ and $x^2 + y^2 + z^2 = r^2$ are orthogonal at every point of intersection.