Dupuy — Math 121 — Homework 07

Instructions Remember to show all of your work to get credit. Please do this assignment on a separate sheet of paper. Remember to show your work.

In these problem we show how double integrals can be used to compute integrals of a single variable.

1. In this problem we will compute the value of

$$\int_0^\infty \frac{\sin(x)}{x} dx.$$

   (a) Show that

$$\int_0^\infty \int_0^\infty e^{-xy} \sin(x) dy dx = \int_0^\infty \frac{\sin(x)}{x} dx.$$

   (Hint: just integrate the left hand side with respect to y on the inside.)

   (b) By switching the order of integration compute

$$\int_0^\infty \int_0^\infty e^{-xy} \sin(x) dy dx.$$

   (c) Compute

$$\int_0^\infty \frac{\sin(x)}{x} dx,$$

   using the previous steps.

2. In this problem we will compute the value of

$$\int_0^\infty e^{-x^2} dx.$$

   (a) Compute

$$\int_0^\infty \int_0^\infty e^{-x^2-y^2} dx dy$$

   by converting to polar coordinates and using that

$$\int_0^\infty \int_0^\infty e^{-x^2-y^2} dx dy = \lim_{a\to\infty} \iint_{D_a} e^{-x^2-y^2} dA$$

   where

$$D_a = \{(x, y) : x^2 + y^2 \leq a^2 \text{ and } x \geq 0 \text{ and } y \geq 0\}.$$

   (b) Find the value of $I = \int_0^\infty e^{-x^2} dx$.

   (Hint: $I^2 = \left(\int_0^\infty e^{-x^2} dx\right) \left(\int_0^\infty e^{-y^2} dy\right) = \int_0^\infty \int_0^\infty e^{-x^2-y^2} dx dy$)