Dupuy — Math 121 — Homework 08

Instructions Remember to show all of your work to get credit. Please do this assignment on a separate sheet of paper. Remember to show your work.

1. Find the Jacobian and Jacobian determinants of the following functions:

   (a) $\vec{F}(r,\theta) = (r\cosh(\theta), r\sinh(\theta))$ we recall that

   $$\cosh(\theta) = \frac{e^{\theta} + e^{-\theta}}{2}$$

   $$\sinh(\theta) = \frac{e^{\theta} - e^{-\theta}}{2}$$

   and that these functions satisfy

   $$\frac{d}{d\theta}\left[\cosh(\theta)\right] = \sinh(\theta)$$

   $$\frac{d}{d\theta}\left[\sinh(\theta)\right] = \cosh(\theta)$$

   $$\cosh(\theta)^2 - \sinh(\theta)^2 = 1$$

   `https://en.wikipedia.org/wiki/Hyperbolic_function`

   (b) $\vec{F}(\rho,\theta,\phi) = (a\rho\sin(\phi)\cos(\theta), b\rho\sin(\phi)\sin(\theta), c\rho\cos(\phi))$ where a, b and c are constants.

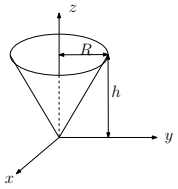
2. (Problem 21 in section 15.9) Find the volume of the region enclosed by the ellipsoid

   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

   where a, b and c are constants.

   (Hint: make a slightly modified version of spherical coordinates where you scale the x, y and z coordinates by a,b and c respectively).

3. Consider a solid cone of height h and radius R made of a material with constant density k as pictured below:

   Compute the moments of inertia for the body using the formulas

   $$I_x = \iiint_E (y^2 + z^2)\rho(x,y,z)dV$$

   $$I_y = \iiint_E (x^2 + z^2)\rho(x,y,z)dV$$

   $$I_z = \iiint_E (x^2 + y^2)\rho(x,y,z)dV$$

where $\rho(x, y, z)$ is the density function.

(You may want to consult the discussion on page 1019 where this is worked out in two dimensions. Note that these values allow us to compute the rotational energy of our body if it is rotating about the x,y or z axis.)

4. Suppose we 3D print a half ball of radius R. Suppose that the plastic it is printed with has density k. Also suppose we set up the printer up so that the fill varies linearly in the y direction starting with 50% fill and ending with a 100% fill. Using the formulas from page 1035 of the textbook answer the following questions:

   (a) What will be the total mass of the half ball?

   (b) What will be the center of mass of the half ball?