

Dupuy — Math 121 — Homework 09

Instructions Remember to show all of your work to get credit. Please do this assignment on a separate sheet of paper. Remember to show your work.

1. Using Mathematica, Matlab or Sage, look-up how to plot vector fields, plot, and print the following vector fields:

(a) $\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$

(b) $\mathbf{G}(x, y, z) = 2xy\mathbf{i} + (y^2 + z^2)\mathbf{j} + 2yz\mathbf{k}$

(Make sure you practice doing some vector field plots by hand before the test!)

2. Let $\vec{r} = (x, y, z)$. Show that

$$U(\vec{r}) = \frac{Q}{4\pi\epsilon_0} \frac{-1}{|\vec{r} - \vec{r}_0|}$$

is a potential for the Coulomb field for a point of charge Q at a point \vec{r}_0 .

3. (a) Let $f = f(x, y, z)$ be a scalar function. Show that

$$\text{curl}(\text{grad}(f)) = 0.$$

- (b) Let \mathbf{F} be a vector field. Show that

$$\text{div}(\text{curl}(\mathbf{F})) = 0.$$

4. Prove the product rules for divergence and curl.

(a) $\nabla \times (f\mathbf{F}) = \nabla f \times \mathbf{F} + f(\nabla \times \mathbf{F})$

(b) $\nabla \cdot (f\mathbf{F}) = \nabla f \cdot \mathbf{F} + f(\nabla \cdot \mathbf{F})$.