Midterm 1 — Dupuy — Math 121 — Fall 2016

Instructions Remember to show all your work to get full credit. Please leave answers in their exact form. This is a closed book test. You may not use a calculator. If you need extra paper let me know.

Last Name, First Name:

Section:

Problem	Possible	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	20	
7	20	
8	10	
Total	100	

1. (10 points) Indicate whether the following expressions make sense (Yes or No).

   (a) $|\vec{a}| \cdot \vec{b}$

   (b) $(\vec{a} \cdot \vec{b}) \times \vec{c}$

   (c) $(\vec{a} \times \vec{b}) \times \vec{c}$

   (d) $(\vec{a} \cdot \vec{b}) \cdot \vec{c}$

   (e) $\vec{a}/|\vec{a}|$.

2. (10 points) Consider the vectors

$$\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k},$$

$$\mathbf{b} = \mathbf{i} + \mathbf{j}.$$

Compute the following

(a) $\mathbf{a} \cdot \mathbf{b}$.

(b) $\mathbf{a} \times \mathbf{b}$.

(c) $\text{proj}_{\mathbf{b}}(\mathbf{a})$.

3. (10 points) Find a parametrization of the line which is the intersection of the two planes:

$$\begin{cases} x - 2y + z = 0, \\ x + y - 2z = 0 \end{cases}$$

4. (10 points) Find the line tangent to the curve parametrized by

$$\mathbf{h}(t) = (e^t + 1)\mathbf{i} + (e^{2t} + 2)\mathbf{j} + (e^{3t} + 3)\mathbf{k}$$

at the point $(2, 3, 4)$.

5. (10 points) Graph the function $f(x, y) = \sqrt{1 - x^2 - y^2}$ for $0 \leq x^2 + y^2 \leq 1$. (Remember that including more information, like traces and labels makes your graph easier to understand.)

6. (20 points) Consider the lines parametrized by

$$\begin{aligned} \alpha(t) &= (t, 1-t, 0), \\ \beta(t) &= (t/2, t/2, 1-t) \end{aligned}$$

(a) Find where the lines parametrized by $\alpha(t)$ and $\beta(t)$ intersect.

(b) Determine an equation for the plane containing the lines parametrized by $\alpha(t)$ and $\beta(t)$.

(You may want to do part (b) on the back to give yourself some space.)

7. (20 points) Assume a and b are non-zero constants.

   (a) Find a parametrization of a line in $\mathbb{R}^3$ passing through the points $(0, 0, 1)$ and $(a, b, 0)$.

   (b) Where does this line intersect the plane $x = 1$?

8. (10 points)

(a) State the definition of the unit tangent, unit normal and unit binormal vectors of a curve parametrized by $\mathbf{r}(t)$.

(b) Show that unit tangent and unit normal vectors of $\mathbf{r}(t)$ are perpendicular. (Hint: take the derivative of both sides of $|\mathbf{T}(t)|^2 = 1$. Use the dot product rule for the LHS.)