Midterm 2 — Dupuy — Math 121 — Fall 2016

Instructions Remember to show all your work to get full credit. Please leave answers in their exact form. This is a closed book test. You may not use a calculator. If you need extra paper let me know.

Name:

Section:

Problem	Possible	Score
1	15	
2	10	
3	10	
4	10	
5	10	
6	15	
7	15	
EC1	10	
EC2	10	
Total	85	
Percentage		

1. (15 points) Compute partial derivatives of the indicated functions

   (a) $\frac{\partial}{\partial y}\left[x^2+y^2+z^2\right]$

   (b) $\frac{\partial^2}{\partial y \partial x}\left[e^{xy}\right]$

   (c) $\frac{\partial}{\partial w}\left[\ln(wx)+1\right]$

2. (10 points) Find the plane tangent to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.

3. (10 points) Determine if the limit exists or not. If the limit exists state its value. If not, explain why.

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}.$$

4. (10 points) Create a level set plot (contour plot) of

$$h(x, y) = x^2 - y^2.$$

make sure to label the level sets for $h(x, y) = -1, h(x, y) = 1$ and $h(x, y) = 0$.

5. (10 points) Let $f(x, y, z) = e^{xy}z$.

(a) In what direction does f have the maximal rate of change at the point $(0, 0, 0)$?

(b) What is the value of the maximal rate of change?

6. (15 points) Evaluate the iterated integral by reversing the order of integration

$$\int_0^1 \int_{2y}^2 e^{x^2}\, dx dy.$$

7. (15 points) Use Lagrange multipliers to maximize the function $f(x, y) = 2x^2 - y^2$ subject to the constraint $2x^2 + y^2 = 2$.

EC1 (10 points) Let c be a constant. Suppose that a level set $\{(x, y) : f(x, y) = c\}$ is parametrized by $\mathbf{r}(t)$, that is

$$f(\mathbf{r}(t)) = c.$$

Show that for every time t, $\mathbf{r}'(t)$ and $\nabla f(\mathbf{r}(t))$ are perpendicular. Make sure to use English sentences if appropriate (I am going to be very strict when grading extra credit so please be precise and explain your steps).

EC2 (10 points) A bell curve with mean μ and standard deviation σ (both constants) is given by

$$g(x) = \frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

Show that

$$\int_{-\infty}^{\infty} g(x)dx = 1.$$

(Half credit will be given if you assume the value of $\int_{-\infty}^{\infty} e^{-x^2}dx$, which you can deduce from a result in Problem 2 of the written part of Homework 07. If you derive the value of $\int_{-\infty}^{\infty} e^{-x^2}dx$ and then use it in your computation you will get full credit.)