Midterm 3 — Dupuy — Math 121 — Fall 2016

Instructions Remember to show all your work to get full credit. Please leave answers in their exact form. This is a closed book test. You may not use a calculator. If you need extra paper let me know. You will be marked off for floating expressions and equalities not related to the problem. You can potentially be marked off for being vague or imprecise.

Name:

Section:

Problem	Possible	Score
1	10	
2	10	
3	10	
4	15	
5	15	
6	15	
7	15	
8	10	
EC1	10	
EC2	10	
Total	100	
Percentage		

1. Determine whether the following operations make sense. In what follows $\mathbf{F} = \mathbf{F}(x, y, z)$ and $\mathbf{G} = \mathbf{G}(x, y, z)$ are vector fields and $f = f(x, y, z)$ a scalar function. (simply say Yes or No).

   (a) $\text{div}(\mathbf{F} \times \mathbf{G})$

   (b) $\text{curl}(\text{curl}(\mathbf{F} \times \mathbf{G}))$

   (c) $\text{curl}(\text{div}(\mathbf{F}))$.

   (d) $\text{div}(f)$

   (e) $\text{curl}(\nabla f)$

2. Consider the vector field $\mathbf{F} = y^2\mathbf{i} + x^2\mathbf{j} + 0\mathbf{k}$

   (a) Compute curl($\mathbf{F}$).

   (b) Compute div($\mathbf{F}$).

3. Let $\vec{F} = (y^2, x^2)$.

   (a) Sketch $\vec{F}$.

   (b) Determine if the following vector field has a potential. If it does, find the potential.

4. Using a triple integral compute the volume of a ball of radius R. I don't care what coordinates you use. Show all of your work.

5. Compute

$$\int_C 2x^2 dx + 2y^2 dy,$$

where C is a the oriented curve parametrized by

$$\vec{r}(t) = (\cos(t)^9(1 - 5\sin(t)^5), \sin(t)^{20} + 2\sin(t)^4\cos(t))$$

for $t \in [0, \pi]$. (Hint: $\vec{r}(0) = (1, 0)$ and $\vec{r}(\pi) = (-1, 0)$)

6. Compute the following contour integral:

$$\frac{1}{2}\oint_C -y dx + x dy$$

where C is a full counterclockwise circle of radius R.

7. Evaluate $\int_T f(x, y, z)dV$ where $f(x, y, z) = 7y$ and T is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 1$.

8. State the fundamental theorem of line integrals. (Be precise).

EC1 Prove the fundamental theorem of line integrals. (Be precise).

EC2 State an prove Poincare's Theorem on the existence of a potential in two dimensions. (To get credit you need to be precise. You may use things we proved in class.)