Dupuy — Complex Analysis — Spring 2017 — Homework 04

Master's Level

1. (Whittaker and Watson, 6.24,3) If $-1 < z < 3$ then

$$\int_0^\infty \frac{x^z}{(1+x^2)^2} dx = \frac{\pi(1-z)}{4\cos(\pi z/2)}$$

2. (Whittaker and Watson, 6.21, Example 4) Let $a > b > 0$ be real numbers. Show that

$$\int_0^{2\pi} \frac{d\theta}{(a+b\cos(\theta))^2} = \frac{2\pi a}{(a^2-b^2)^{3/2}}$$

3. (Whittaker and Watson, 6.23, 2) If $a > 0$ and $b > 0$ show that

$$\int_{-\infty}^\infty \frac{x^4 dx}{(a+bx^2)^4} = \frac{\pi}{16a^{3/2}b^{5/2}}$$

4. (Whittaker and Watson, 6.22, 1) Show that if $a > 0$ then

$$\int_0^\infty \frac{\cos(x)}{x^2+a^2} dx = \frac{\pi}{2a} e^{-a}.$$

Ph.D. Level

5. (Whittaker and Watson, 6.22) If the $\operatorname{Re} z > 0$ then

$$\int_0^\infty (e^{-t} - e^{-tz}) \frac{dt}{t} = \log z$$

6. (Whittaker and Watson, 6.24,2) If $0 \leq z \leq 1$ and $-\pi < a \leq \pi$ then

$$\int_0^\infty \frac{t^{z-1}}{t+e^{ia}} dt = \frac{\pi e^{i(z-1)a}}{\sin(\pi z)}$$

7. (Whittaker and Watson 6.24, 1, pg118) If $0 < a < 1$ show that

$$\int_0^\infty \frac{x^{a-1}}{1+x} dx = \pi \csc a\pi$$

8. (Whittaker and Watson, 6.24, 4)Show that if $-1 < p < 1$ and $-\pi < \lambda < \pi$ we have

$$\int_0^\infty \frac{x^{-p} dx}{1+2x\cos(\lambda)+x^2} = \frac{\pi}{\sin(p\pi)} \frac{\sin(p\lambda)}{\sin(\lambda)}$$

9. (Whittaker and Watson, 6.21, Example 3) Let n be a positive integer. Show that

$$\int_0^{2\pi} e^{\cos(\theta)} \cos(n\theta - \sin\theta) d\theta = \frac{2\pi}{n!}$$