The Oort Conjecture and the local lifting problem

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Setup

Notation:

- *p* a prime number.
- *k* an algebraically closed field of characteristic *p*.
- G a finite group.
- Let *G* act faithfully on *k*[[*z*]] by (*z*-adically) continuous *k*-algebra automorphisms.
 - Note: the action of each g ∈ G is specified by g(z), which is a power series of the form

$$a_1z+\sum_{i=2}^{\infty}a_iz^i,$$

and $a_1 \neq 0$.

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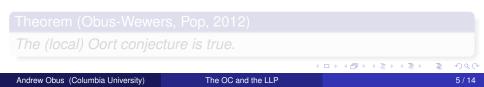
- The local lifting problem (LLP) asks: Does there exist a finite extension *R*/*W*(*k*), and a local action of *G* on *R*[[*Z*]] by *R*-algebra automorphisms, which lifts the given action on *k*[[*z*]]?
- If any such *R* exists, the local *G*-action is said to *lift to characteristic zero*.
- The *local Oort conjecture*: If *G* is *cyclic*, the local *G*-action lifts to characteristic zero.
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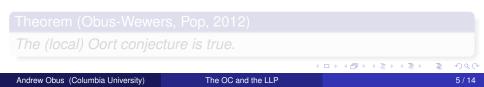
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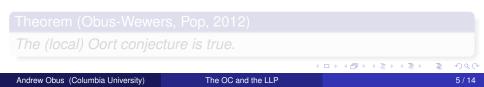
- Let X/k be a smooth, projective curve, with a group action H → Aut(X).
- We say *X* (with *H*-action) lifts to characteristic zero if there exists R/W(k) finite, X_R a smooth projective *R*-scheme with special fiber *X*, and an *H*-action on X_R reducing to the given *H*-action on *X* on the special fiber.
- Local-Global Principle: X (with *H*-action) lifts to characteristic zero iff for each $x \in X$, the action of the stabilizer H_x on $\hat{\mathcal{O}}_{X,x} \cong k[[z]]$ lifts to characteristic zero.
- Oort Conjecture (1987): If *H* is cyclic, *X* (with *H*-action) lifts to characteristic zero. Equivalent to local Oort conjecture.



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- Know $k[[z]]^G \cong k[[t]]$, and $R[[Z]]^G \cong R[[T]]$ for a lift.
- Rephrase local lifting problem: When does a *G*-Galois extension k[[z]]/k[[t]] lift to a *G*-Galois extension R[[Z]]/R[[T]]?
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The $G = \mathbb{Z}/p$ case (Kummer-Artin-Schreier theory)

For G ≃ Z/p, local Oort conjecture is due to Oort-Sekiguchi-Suwa (1988).

 In characteristic p, all extensions can given by taking the normalization of k[[t]] in

$$k((t))[y]/(y^{p}-y-t^{-m}),$$

for some $m \in \mathbb{N} \setminus p\mathbb{N}$.

- Take R = W(k)[ζ_p]. One finds an explicit Kummer extension of R[[T]] giving the correct reduction.
- Form is suggested by the *Kummer-Artin-Schreier theory*, which gives an exact sequence interpolating the standard Kummer and Artin-Schreier exact sequences.

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- Idea is similar, but equations are significantly more complicated.
- Instead of the Kummer-Artin-Schreier theory, one uses the *Kummer-Artin-Schreier-Witt theory* (AKA *Sekiguchi-Suwa* theory).
- Gives a group scheme interpolating between Kummer and Artin-Schreier-Witt exact sequences.
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Proof of the local Oort conjecture: the work of Obus-Wewers

Theorem (Obus-Wewers, 2012)

The local Oort conjecture is true, given a certain condition on the higher ramification filtration for k[[z]]/k[[t]].

- Condition is vacuous when $G \cong \mathbb{Z}/p^3$.
- Condition satisfied when k[[z]]/k[[t]] has "no essential ramification." That is, for each upper jump u_i, have u_{i+1} < pu_i + p.
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- Main Idea: Given a Z/pⁿ-extension k[[z]]/k[[t]], instead of trying to write down a lift, instead try to write down *some* Kummer extension of R[[T]] that reduces to a Galois extension of k[[t]].
 - In general, one gets something inseparable, but can measure the inseparability (Kato's differential Swan conductor).
 - Given a guess where the inseparability is small enough, can make a deformation to reduce it.
 - Can show that this deformation process terminates using harmonic function theory on rigid-analytic spaces.
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