

Course Objectives and Problems  
Math 1055 — Spring 2024— Dupuy

**WARNING:** Prior to being assigned a quiz date, homework problems are subject to change.

**Propositions**

1. I understand truth tables (§1.1: 1a, 1c, 2b, 2d, 2e).

Also, I asked ChatGPT to prove de Morgan's Law that

$$\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q).$$

This is what it produced:

*ChatGPT Generated Proof.* To show that  $\neg(P \vee Q)$  is logically equivalent to  $(\neg P \wedge \neg Q)$ , we can use the following truth table:

$P$	$Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$(\neg P \wedge \neg Q)$
T	T	F	F	F	F
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

As we can see from the truth table, in all cases  $\neg(P \vee Q)$  and  $(\neg P \wedge \neg Q)$  have the same truth value, therefore they are logically equivalent. □

Is ChatGPT correct? Where is the error if at all?

2. I understand how to read logic and convert to english (§1.1: 7, 12, 15)
3. I understand how to use basic properties of the integers to prove things. (§1.2: 1,2,3,4, §2.1: 1)
4. I understand summation notation, can state and prove the geometric sum formula, and can state and prove the difference of powers formulas. (The quiz is going to ask you to state and prove the geometric sum formula).

Also, I asked ChatGPT to state and prove the Geometric Sum Formula. This is what it did (I helped with the LaTeX a little).

*ChatGPT Solution.* The geometric sum formula states that the sum of the first  $n$  terms of a geometric sequence  $a, ar, ar^2, \dots$  is equal to:

$$\frac{a(1 - r^n)}{(1 - r)}$$

where  $a$  is the first term of the sequence,  $r$  is the common ratio, and  $n$  is the number of terms in the sum.

This can be proven by using the formula for the sum of an arithmetic series, which states that the sum of an arithmetic series is equal to the average of the first and last term, multiplied by the number of terms.

For a geometric series, the first term is  $a$ , and the  $n$ th term is  $a * r^{(n-1)}$ . So the sum of the first  $n$  terms is:

$$S_n = a + ar + ar^2 + \dots + ar^{(n-1)}$$

This can be rewritten as:

$$S_n = a(1 + r + r^2 + \dots + r^{(n-1)})$$

Now, using the formula for the sum of a geometric series:

$$S_n = a(1 - r^n)/(1 - r)$$

Therefore, the geometric sum formula has been proven. □

Is ChatGPT correct? Where are the errors if there are any?

### Proofs

1. I understand direct proof techniques
  - (a) I understand counter-examples
  - (b) I understand how to prove existential statements:
  - (c) I understand the basic concepts surrounding the integers.
  - (d) I understand basic properties of real numbers and how to use them in proofs.
  - (e) I understand proofs by cases
  - (f) I understand how to prove biconditional statements
  - (g) I understand what uniqueness is and how to use it and how to prove it  
(§2.1: 3, 5a-i, 7, 15, 17, 21)
2. I understand indirect proof techniques
  - (a) I know how to prove a conditional statement.
  - (b) I know how to use a conjunctive hypothesis.
  - (c) I know how to use an existential hypothesis.
  - (d) I know how to use a universal hypothesis.
  - (e) I know how to use a contrapositive in a proof.  
(§2.2: 1,2,3, 6a, 6b, 7, 8, 9, 10, 12 (use a difference of powers), 13 )
3. I know how to give a proof by contradiction (§2.3: 1,2,3,4)

### Induction

1. I know how to do proofs by weak induction. (§3.1: 2,3,4,8,9,11,12,15,16)
2. I know how to prove the binomial theorem (§3.1: 18,19)
3. I know how to do proofs by strong induction. (§3.2: 1, 2)
4. I can prove the fundamental theorem of arithmetic.

### Sets

1. I understand the different notations for sets. (§4.1: 1a-k)
2. I understand how to prove something is or is not an element of a set.
3. I know how to work with the empty set.
4. I now how to show one set is a subset of another.
5. I know how to show two sets are equal (§4.1: 5, 6)
6. I understand the basic set operations (intersection, union, complement) (§4.2: 1a-i, 2, 3, 5, 8, 11\*, 12, 15, 16, 17, 22, 23, 24)
7. I understand indexed families of sets and how to work with them. (§4.2: 1,3,5,6,9)

### Functions

1. I understand functions, domains, codomains, images, and preimages. (§5.1: 1, 2, 4, 5, 7)

2. I understand injectivity, surjectivity, and bijectivity (§5.3: 1, 2, 7; §5.4: 1a-o, 10).
3. I understand function composition. (§5.2: 1, 4)
4. I understand functions on sets (§5.5: 2,3,4,8,10,11)

**Arithmetic**

1. I understand the well-ordering principal and I can use it in a proof. (§6.1: 7, 8 )
2. I understand the division algorithm and can use it in a proof by cases.(§6.1: 3)

**Relations and Equivalence Classes** This will be a jumble of §6.4, §6.5, §7.1, §7.2, §7.3

1. I understand what a relation is. (§7.1: 1)
2. I understand reflexivity, transitivity, and symmetry of relations. (§7.2: 2, 6)
3. I understand equivalence classes (§7.2: 4)
4. I understand what a quotient set it and what a quotient map is. (§7.2: 9, §7.3: 3)
5. I understand what it means for a function to be well-defined. (§7.2: 10, §6.5: 9)
6. I understand modular arithmetic. (§6.4:1, 2, 5, 9 [this is a classic, you need to use that  $p$  divides  $\binom{p}{j}$  for  $1 \leq j \leq p - 1$ .])

**Cardinality** The following covers §8.1, §8.2, §8.3,

1. I know the difference between a finite and infinite set. (§8.1: 1, 3, 9)
2. I know the difference between a countable and uncountable set. (§8.2: 2, 3,5, 12)
3. I know the basic examples of countable and uncountable sets. (§8.3: 1, 3, 10, 14, 17, 18)