MATH 2055 A — Fundamentals of Mathematics

Taylor Dupuy, University of Vermont

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Office Hours TBA. Announced in class and posted on the course webpage during the first week.

Course Meeting Times MWF 9:40–10:30, Kalkin 001. CRN 92912. 3 credits. Classes run September 9 through December 11, 2026 — 37 meetings.

Final Exam There is no final exam in this course. The Registrar has assigned this course a final exam slot, Monday, December 14, 2026, 10:30 AM–1:15 PM. **We will not use it.** Nothing is due, nothing is graded, and nothing meets on December 14. Please read this now rather than in December: you may book travel for the finals period without checking with me.

Book Daniel J. Velleman, *How To Prove It: A Structured Approach*, 3rd edition, Cambridge University Press, 2019. ISBN 978-1-108-42418-9.

Companion Text Daniel J. Velleman, *How To Prove It with Lean*. Free online at https://djvelleman.github.io/HTPIwL/. It is free and is not required reading; see the section on Lean below.

Content Chapters 1–6 and Chapter 8 of Velleman: sentential logic, quantificational logic and sets, proof strategies, relations, functions, mathematical induction, and infinite sets. Chapter 4 is taught only as far as equivalence relations require, and congruence modulo m is introduced with them in §4.5. **Chapters 1–2 are resequenced** — see the section on reading order below — but from Chapter 3 onward we read the book in Velleman's own order, induction included, and then Chapter 8, which is where Velleman puts it. Chapter 7 is skipped. Not covered: §4.4, §5.4, §6.2, §6.5 and Chapter 7.

Chapter 8 has the last three meetings of the term and it is a tight fit; see the note under the schedule for what I expect to finish and what may not happen.

Prerequisites Enforced: MATH 1234 or MATH 1242.

Strongly recommended as a co-requisite: a course in vector calculus. It is not required, nothing here assumes it, and you will not be at a disadvantage without it. But students who are seeing multivariable material at the same time as this course tend to get more out of both. Proof-writing needs something to be about, and vector calculus supplies a steady stream of objects, definitions and statements worth proving things about. The two courses reinforce each other: the proofs give the calculus its reasons, and the calculus gives the proofs their subject matter.

Grading **Your grade is the average of the top 80% of your quiz scores. That is the entire grade.** There are no midterms, there is no final exam, and there is no graded homework. There are 11 quizzes, so the top 80% means **your two lowest scores are dropped** and the remaining nine are averaged. **Each quiz is graded out of 10 points**, and the number is printed at the top of every quiz sheet, so your grade is the average of your nine best scores out of 10, expressed as a percentage. Letter grades are assigned on the usual scale (A: 90–100, B: 80–89, C: 70–79, D: 60–69, F: below 60), with plus/minus grades at the top and bottom of each band. I reserve the right to curve upward, never downward.

Quiz Dates Every Friday that we meet, except September 11. In order: September 18 and 25; October 2, 16, 23 and 30; November 6, 13 and 20; December 4 and 11. That is 11 quizzes.

The quizzes are the course

There is one quiz every Friday we meet, taken in class, on paper, by hand. It is a single question and it takes about ten minutes. The question comes from the material of the preceding week, and it is written for this course — in the spirit of Velleman's exercises, but not copied out of the book.

Eleven quizzes are given, each out of 10 points. **The top 80% of them — the highest nine — are averaged, and that average is your grade.** Your two lowest scores are dropped. If your eleven scores were 8, 9, 6, 10, 7, 0, 9, 8, 10, 7 and 9, the 0 and the 6 come off, the remaining nine total 77, they average 8.56 out of 10, and your grade is an 85.6 — a B. A missed quiz scores zero and is one of the scores eligible to be dropped, exactly like a bad score on a quiz you did sit.

There is no quiz on Friday, September 11. That is only the second meeting of the term, and a single class is not much to be quizzed on.

No redos, no makeups, no documentation

Please read this section carefully. It is the part of the syllabus that surprises students most often, and I would much rather you not be surprised.

If you are sick, please stay home. You do not need to send me a doctor's note, and there is no need to apologise. Please do not come to class unwell in order to protect a score. The two dropped quizzes exist for exactly this — for the flu, for the funeral, for the flat tire, for the morning that simply goes wrong. They are yours to use, you do not need my permission, and you never have to tell me why. Using one costs you nothing at all.

What I would like you to understand before the term begins is that **those two drops are the whole safety net.** There are no redos, no makeup quizzes, no alternate sittings, and no exam later in the term that could absorb a missed Friday. If a third Friday goes wrong, that third zero is averaged into your grade, and I will not be able to make an exception — not because I doubt the reason and not for any lack of sympathy, but because this is the other half of a trade I have made on purpose. I never ask any of you to explain or document an absence, and in exchange I do not weigh one student's reason against another's. For a class this size I think that is the kinder arrangement, but it only works if it holds for everyone.

So: two free misses, no questions asked, and please keep them in reserve for a week when you really need them. If the term starts to get away from you, come and talk to me early. I cannot change a score, but there is a great deal else I can usually help with.

Two things sit outside this rule, and both are University policy rather than mine. **Accommodations arranged through Student Accessibility Services** come first: if you have an SAS accommodation

letter, bring it to me in the first two weeks and we will put in place whatever it calls for — extended time, a separate room, or an alternate sitting — for every quiz in the term. Nothing in this section overrides it. The second is **Religious Holidays**. Both are described in the University Policies at the end of this syllabus.

Homework and practice problems

There is no graded homework in this course. Practice problems will be posted and worked in class, but they are not collected and they do not count toward your grade. The reason is not that work done outside of class does not matter — it is very nearly all that matters — but that written work done outside of class can no longer be reliably attributed to the student who submitted it, so grading it would be measuring the wrong thing. Everything that counts in this course is therefore written in class, by hand, without notes or devices.

Please do the practice problems anyway. In this course, more than in most, they are the whole preparation. Writing proofs is a physical skill in the way that playing an instrument is a physical skill: reading someone else's proof and agreeing with it is not the same activity as producing one on a blank page, and only one of those two is what Friday asks for.

How much time this takes. The usual rule of thumb is about three hours of work outside of class for every hour spent in it. This course meets three hours a week, so plan on roughly nine hours a week of reading, problems and writing — an hour and a half a day, most days of the week. That is not a threat and it is not a formula for a grade. It is a description of what the students who feel comfortable on Friday morning are actually doing during the week.

The suggested problems. Lists of recommended problems are kept for each part of the course and posted on the course webpage,
https://tdupu.github.io/logic/2026/math2055.html
Quiz questions are written to be **similar to those problems, but not identical to them.** What I am asking is that you can do problems *of that kind*: the skill is meant to transfer, and a memorised solution does not transfer. If you can work a recommended problem on a blank page without looking at anyone's answer, Friday will feel familiar.

Work with other people. Find two or three classmates in the first couple of weeks and set a standing time to work problems together. Explaining a proof out loud to somebody who is not yet convinced is the fastest way to discover whether you really have one, and a study group is much easier to start in September than in November.

You are also very welcome to ask an AI for help with a practice problem — I recommend it, and there is more about this in the next section. I will ask one thing of you about how you use it, and it is the only thing I will ask: **read the problem and make a genuine attempt at it before you look up an answer or ask a model.** Give it ten or fifteen minutes, put something on the page, get properly stuck. The value is in the attempt. A solution you have read is not a skill you have, and the distance between those two is exactly what Friday measures.

AI tools

I **strongly recommend** that you use AI coding and reasoning tools while you study, and two in particular: **Claude Code** and **Codex**. Learn to drive them well. They are patient, they are available at two in the morning, and being able to work with them is going to matter in whatever you go on to do.

Since I am recommending them, you should know that I use them. **This syllabus, the course webpages and the problem sets for this course were generated with Claude Opus 5 in a Cowork harness.**

The decisions, the sequencing and the policies are mine, and I have read and edited every line of what you are holding — but the drafting was done with a model, and it would be strange to recommend a tool to you while quietly concealing that I use it myself.

If you would like help with any of this, please ask me. Installing these tools, prompting them, working out what they are and are not good at — I am glad to help, in office hours or after class, and it is not an off-topic question.

None of this is in tension with the fact that no work outside of class is graded. The two ideas fit together. Nothing you do outside of class counts toward your grade, and the reason is not that these tools are forbidden; it is that because they exist and are good, work produced outside a classroom no longer tells me — or you — very much about what you can do on your own. So I do not grade it, and I have no interest in policing it. What I grade is the ten minutes on Friday, on paper, by hand. The rest of the week is yours to learn with by whatever means work for you, and AI is one of the means I recommend.

The one boundary is the quiz itself, and it is not really a rule about AI: quizzes are closed-book and closed-device and are your own work, as the Academic Integrity section below says. That is a rule about the quiz. It is not a rule about how you study.

Reading order: Chapters 1–2 are resequenced

Velleman braids logic and set theory together through Chapters 1 and 2 — sets are his worked example for the connectives, so his sections on sets sit inside his chapters on logic. This course pulls them apart. That is the one deliberate departure from the book's order: from Chapter 3 onward we read Velleman straight through — Chapter 3, then 4, 5, 6 and 8, skipping Chapter 7 — and induction stays in Chapter 6, where he puts it.

Chapters 1–2 are taught as three units, logic first and sets last.

Unit I — Propositional Logic §1.1, §1.2, §1.5.

Unit II — Predicate Logic the first part of §1.3, then §2.1, then the logic half of §2.2. Membership $\in$ appears here as a bare primitive, only because bounded quantifiers $\forall x \in A$ are an abbreviation; nothing else about sets is needed yet.

Unit III — Sets the rest of §1.3, §1.4, the set half of §2.2, and §2.3.

One consequence to plan around: the ten equivalence laws are learned in §1.2 and then go unused for two or three weeks, until §1.4 in Unit III. You get a laws reference sheet in week 1 so that Unit III is a lookup rather than a re-learn.

Because the book and the course disagree about Chapters 1–2, a section-by-section map of what we read when is maintained at
https://tdupu.github.io/logic/2026/reading-order.html
That page also lists which exercises to skip. A handful of Velleman's exercises depend on §4.4 (ordering relations), which this course does not cover, and a few more in Chapters 1–2 depend on material the resequencing pushes later. Check it before reading ahead.

Lean (ungraded)

Lean is a proof assistant: you write a proof and the computer checks it, line by line, and tells you exactly where it is not yet convinced. Two of our class meetings are Lean days — **Wednesday October 28** (after

Chapter 3) and **Friday December 11** (after Chapters 5 and 6). **These are ordinary class meetings.** They are not optional, not opt-in and not extra: we do Lean together in the room on those days, the same way we do anything else. Each is scheduled *after* we have already done the corresponding material on paper, so that Lean is a second look at something you already know rather than a new subject. December 11 is a Lean day only in part: it is also the last day of class, it carries Quiz 11, and §8.3 is scheduled on it. Expect a demonstration that day rather than a full working session. The companion text is Velleman's *How To Prove It with Lean*, free at https://djvelleman.github.io/HTPIwL/, and a walkthrough with setup instructions lives at https://tdupu.github.io/lean/.

Two things to do before Wednesday October 28. First, **make a free GitHub account** if you do not already have one, at https://github.com/signup. It takes two minutes, it costs nothing, and you need one to open the Lean package in a browser tab, which is what we will be doing in the room. Second, **bring a laptop that day** if you have one. If you do not have a laptop, come anyway and share with a neighbour — working in pairs is genuinely better — but the more laptops in the room the better the day goes. There is nothing to install beforehand.

There is no third Lean day. All 37 meetings are assigned and there is nowhere to put one. Lean day 2 does now fall after induction rather than before it, so induction is at least within reach on December 11 — but that meeting is shared three ways, so please plan on the Lean treatment of induction living on the walkthrough page instead, written to be read on your own. I am glad to work through it with anyone who wants it in office hours.

Nothing about Lean is ever graded or quizzed. It cannot help your grade and it cannot hurt it. Since the grade is nothing but quiz scores, and no quiz question will ever be about Lean, there is no surface on which Lean could reach your transcript at all. What *is* optional is everything outside those two meetings: the walkthrough pages, installing Lean on your own machine, working exercises on your own time, and the no-credit completion badge, which is awarded after a short conversation in office hours and confers no academic credit of any kind. You are not required to install anything or to do any Lean outside class. Details are on the webpage above. Bring a laptop on the two Lean days if you have one; if you do not, share one, which is better anyway.

Attendance

I do not take attendance and there is no participation grade. That said, the quizzes come out of what happens in the room and they happen *in* the room, so it is genuinely hard to do well in this course without being here. If you must miss class for a university-sanctioned event — an intercollegiate athletic competition, for instance — please let me know in advance so we can plan around it.

Week-by-week schedule

This schedule is a plan, not a contract; topics may shift by a meeting or two. The quiz dates will not move. Fall Recess (Friday, October 9) and Thanksgiving recess (November 23–27) are already removed below. The term is 37 meetings.

Wk	Meeting	Topic (Velleman)	Events
1	Wed Sep 9	Course intro; §1.1 logical connectives	First day of class; Unit I begins
	Fri Sep 11	§1.1–1.2	No quiz
2	Mon Sep 14	§1.2 the equivalence laws	Add/Drop, Pass/No Pass, Audit deadline

Wk	Meeting	Topic (Velleman)	Events
	Wed Sep 16	§1.5 conditional and biconditional	
	Fri Sep 18	§1.5	**Quiz 1**
3	Mon Sep 21	§1.3 predicates, free and bound variables; $\in$ introduced	Unit II begins
	Wed Sep 23	§2.1 quantifiers	
	Fri Sep 25	§2.1 nested and bounded quantifiers; $\forall\exists$ versus $\exists\forall$	**Quiz 2**
4	Mon Sep 28	§2.2 quantifier equivalences (logic half)	
	Wed Sep 30	§1.3 set-builder notation; §1.4 operations on sets	Unit III begins
	Fri Oct 2	§1.4; §2.2 set half	**Quiz 3**
5	Mon Oct 5	§2.3 families, power sets, Russell's paradox	
	Wed Oct 7	§3.1 proof strategies	Chapter 3 begins
	Fri Oct 9	*Fall Recess — no class*	
6	Mon Oct 12	§3.2 negations and contradiction	
	Wed Oct 14	§3.3 proofs involving quantifiers	
	Fri Oct 16	§3.3; the order of mixed quantifiers	**Quiz 4**
7	Mon Oct 19	§3.4 conjunctions and biconditionals	
	Wed Oct 21	§3.5 disjunctions and proof by cases	
	Fri Oct 23	§3.6 existence and uniqueness	**Quiz 5**
8	Mon Oct 26	§3.7 ε–δ limit proofs	
	Wed Oct 28	Lean	**Lean day 1** — bring a laptop
	Fri Oct 30	§4.1–4.2 Cartesian products, relations	**Quiz 6**; Chapter 4 begins
9	Mon Nov 2	§4.2 inverse and composition	Last day to withdraw
	Wed Nov 4	§4.3 reflexive, symmetric, transitive	
	Fri Nov 6	§4.5 equivalence relations; congruence mod m	**Quiz 7**
10	Mon Nov 9	§4.5 equivalence classes, partitions, $\mathbb{Z}/\equiv_m$	
	Wed Nov 11	§5.1 functions; well-definedness on a quotient	Chapter 5 begins
	Fri Nov 13	§5.2 one-to-one and onto	**Quiz 8**
11	Mon Nov 16	§5.3 inverses of functions	
	Wed Nov 18	§5.5 images and inverse images	
	Fri Nov 20	§6.1 proof by mathematical induction	**Quiz 9**; Chapter 6 begins
	Nov 23–27	*Thanksgiving recess — no class*	
12	Mon Nov 30	§6.1	
	Wed Dec 2	§6.3 recursion	
	Fri Dec 4	§6.4 strong induction, well-ordering, division algorithm, irrationality of $\sqrt{2}$	**Quiz 10**
13	Mon Dec 7	§8.1 equinumerous sets	Chapter 8 begins
	Wed Dec 9	§8.2 countable and uncountable sets; Cantor's theorem	
	Fri Dec 11	§8.3 the Cantor–Schröder–Bernstein theorem	**Quiz 11**; **Lean day 2** (in part); last day of class

Chapter 8 is a tight fit, and I would rather say so in September than in December. Infinite sets get three meetings — December 7, 9 and 11. §8.1 (equinumerous sets) and §8.2 (countable and uncountable sets, Cantor's theorem) are the priority, and those two are what I intend to finish. §8.3, the Cantor–Schröder–Bernstein theorem, is a stretch goal rather than a promise: it sits on the last day of class and it is the first

thing that goes if the term has run even a meeting long. If we do not get to it, I will post a written treatment and I am glad to work through it with anyone in office hours.

Spare capacity. The table above assigns all 37 meetings, and the slack is thin. The one genuinely soft meeting is **Wednesday, October 28** (Lean day 1). It is a real class meeting and I intend to spend it on Lean. But Lean is the one thing on the calendar that is never quizzed and that has a written home outside the room, so it is also the one meeting I can spend on mathematics instead if we are running behind by late October, with the Lean material moving to the walkthrough page and to office hours. If that happens I will say so in class; either way there is class that day. After that the give is all at the end of the term, in this order: §8.3 goes first, then §8.2 compresses into a single meeting with §8.1. **Nothing in Chapters 1–6 gives.** Note that December 11 is carrying more than any other day in the term — Quiz 11, the second Lean day, and §8.3 — and if all three will not fit, §8.3 has the strongest claim on the time.

Reading days are December 12, 13 and 16. The final exam period is December 12–18. Neither concerns this course: **there is no final exam**, and December 11 is the last day you need to be here.

University policies

Academic Integrity All work in this course is governed by UVM's Code of Academic Integrity, https://www.uvm.edu/policies/student/acadintegrity.pdf. In practice, in this course, that means the Friday quizzes are your own work, written without notes, devices, or assistance.

Code of Student Rights and Responsibilities Students are responsible for knowing the Code, https://www.uvm.edu/policies/student/studentcode.pdf.

Student Accessibility Services UVM is committed to full access for students with documented disabilities. Contact Student Accessibility Services (SAS), A170 Living/Learning Center, 802-656-7753, access@uvm.edu. If you have an accommodation letter, please arrange to meet with me within the first two weeks of the semester so that accommodations can be put in place well before the first quiz on September 18.

Religious Holidays Students have the right to practice the religion of their choice. If you will miss class or a quiz for a religious holiday, submit your documented schedule of absences to me in writing by the end of the second full week of classes. This is University policy and it takes precedence over the no-makeups rule above. **A quiz missed under it is excused outright: it is removed from the count, and your grade is the average of the top 80% of the quizzes that remain** — so your two lowest remaining scores are still dropped, and the excused quiz is not one of them. Concretely: with one quiz excused, ten quizzes remain, the two lowest of those ten are dropped, and the best eight are averaged. There is no makeup sitting; the arithmetic does the work instead.

Attendance and Athletic Absence Students representing the University in a sanctioned event should provide me with their schedule of expected absences in advance. See the attendance policy in the Code of Student Rights and Responsibilities above.

Course Evaluation All students are expected to complete an evaluation of the course at its conclusion. Evaluations are anonymous and confidential, and I do not see them until after grades are submitted. I read them carefully and they change how I teach this course.