Derivatives

Dupuy—Math 150—Summer 2008

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You're expected to think a little bit.

1. What is a derivative? How was it defined? (Use a couple of sentences.)
2. What are the rules for finding the derivatives of
   (a) $f(x) = a$, where a is a constant.
   (b) $f(x) = x^n$ where n is a natural number.
   (c) $f(x) = g(x) + h(x)$.
   (d) $f(x) = a \cdot g(x)$ where a is a constant and $g(x)$ is another function.
   (e) $f(x) = mx + b$, where m and b are constants.
3. Compute the derivatives of the following functions:
   (a) $f(x) = x^4 + x^3 + x^2 + x + 1$.
   (b) $g(x) = x^2 + 1$.
   (c) $h(x) = x^3 + 2x$.
   (d) $f(x) = (x^2 + 1)(x^3 + 2x)$.
   (e) $g(x) = (5x^4 + 2x^2)$
   (f) $h(x) = (x + 1)$.
   (g) $f(x) = (5x^4 + 2x^2)(x + 1)$
   (h) $g(x) = x^4$
   (i) $h(x) = x^3$
   (j) $f(x) = x^7$.
4. In the above problems do you see a pattern with finding the derivatives of $f(x) = g(x)h(x)$? For this problem let $f(x) = g(x)h(x)$ where $g(x) = x^2 + 1$ and $h(x) = 2x^3 + x$.
   (a) Multiply out $g(x)h(x)$ to get an expanded form of $f(x)$.
   (b) Compute $f'(x)$.
   (c) Compute $g'(x)$.
   (d) Compte $h'(x)$.
   (e) Verify that $f'(x) = g'(x)h(x) + g(x)h(x)$.
5. The product rule is the following

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x). \tag{1}$$

   Prove this rule. Here are some hints: 1. Write out the difference quotient for $F(x) = f(x)g(x)$, $(\Delta_h F)(x)$. 2. TRICK: Add 0 in the form of $0 = \frac{f(x+h)g(x)}{h} - \frac{f(x+h)g(x)}{h}$ to your difference quotient and then factor the result to look something like the above rule remembering that for any function $G(x)$, $(\Delta_h G)(x)$ is really just $G'(x)$ when we let $h = 0$.

6. Compute the derivative $f(x) = g(x)h(x)$ *using the product rule* when

   (a) $g(x) = x$, $h(x) = 1$.

   (b) $g(x) = x$, $h(x) = x$.

   (c) $g(x) = 3x + 1$, $h(x) = x^2 - 5$

   (d) $g(x) = x^n$, $h(x) = x^m$ where n and m are natural numbers.

7. Try to find a rule for the derivative of $f(x) = x^{-n}$ where n is a natural number. Try doing some examples first.

8. Try to compute the derivative of $f(x) = x^{1/2}$. Recipe: Use the product rule on $f(x)f(x)$ to get one side of the equation (i.e. $(f(x)f(x))' = f'(x)f(x) + f(x)f'(x)$). Then use the fact that $f(x)f(x) = x$ along with out power rule (i.e. $(f(x)f(x))' = (x)' = 1$) to get the other side of the equation. Since we know $f(x) = x^{1/2}$ you can replace $f(x)$ with $x^{1/2}$ where it appeared in the product rule. Finally, solve your equation for $f'(x)$. Put this all together and state your result. Can you derive any other rules like this?