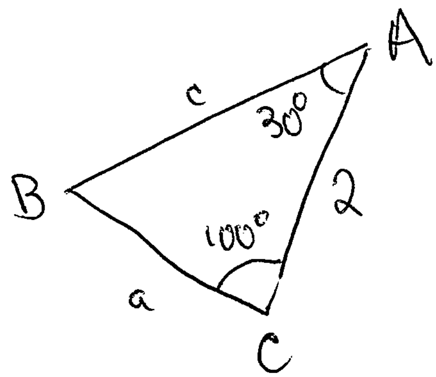
SOLNS HW 3

6.4: 8
6.5: 32, 35
7.1: 2, 38
7.2: 24, 32

6.4: 8



Solve for angle B:

$$A + B + C = 180°$$

$$\Rightarrow 30° + B + 100° = 180°$$

$$\Rightarrow B = 50°$$

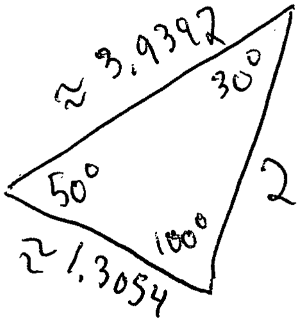
Use law of sines twice to get sides a & c

- $$\frac{\sin B}{b} = \frac{\sin A}{a} \Rightarrow \frac{\sin(50°)}{2} = \frac{\sin(30°)}{a}$$

  $$\Rightarrow a = 2\,\frac{\sin(30°)}{\sin(50°)} \approx 1.3054$$

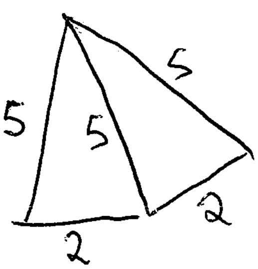
- $$\frac{\sin C}{c} = \frac{\sin B}{b} \Rightarrow \frac{\sin(100°)}{c} = \frac{\sin(50°)}{2}$$

  $$\Rightarrow c = 2\,\frac{\sin(100°)}{\sin(50°)} \approx 3.9392$$



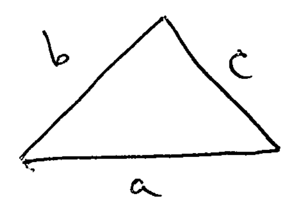
6.5 : 32

Find the area of the figure



call area of this figure A_{fig}

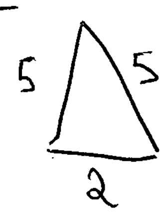
HERON'S FORMULA:



$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where

$$s = \frac{1}{2}(a+b+c).$$



$a = 5$
$b = 5$
$c = 2$

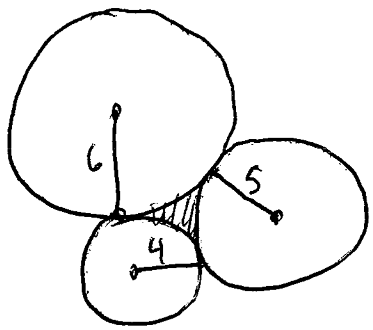
$\Rightarrow \quad s = \frac{1}{2}(5+5+2) = \frac{1}{2}(12) = 6.$

$$\therefore \text{Area of Little } \Delta = \sqrt{6(6-5)(6-5)(6-2)}$$
$$= \sqrt{6 \cdot 1 \cdot 1 \cdot 4}$$
$$= 2\sqrt{6}$$

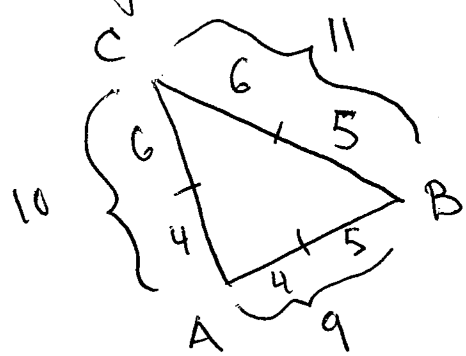
$$A_{fig} = 2\left(\text{Area of Little } \Delta\right) = 2(2\sqrt{6}) = 4\sqrt{6}.$$

6.5 : 35

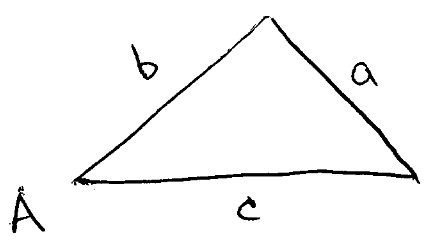
Find the area of the region enclosed by the three circles



• Solve for angles of arc using the triangle



Law of Cosines:



$$b^2 + c^2 - 2bc\cos A = a^2$$

solve formula for cos A in rule

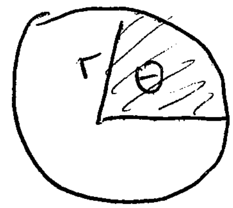
$$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$$

$$\cos A = \frac{11^2 - 9^2 - 10^2}{-2\cdot 9\cdot 10} \Rightarrow A = \cos^{-1}(1/3)$$
$$\approx 1.2309$$
$$= 1/3.$$

$$\cos B = \frac{10^2 - 9^2 - 11^2}{-2\cdot 9\cdot 11} \Rightarrow B = \cos^{-1}(17/33)$$
$$\approx 1.0296$$
$$= 17/33.$$

$$\cos C = \frac{9^2 - 10^2 - 11^2}{-2\cdot 10\cdot 11} \Rightarrow C = \cos^{-1}(7/11)$$
$$\approx .8810$$
$$= 7/11.$$

Area ~~of Δ~~ under Arc:



$A = \frac{\theta}{2} r^2$

Area of triangle: use Heron's formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a+b+c)$ ☆

what I'm going to use to solve the problem

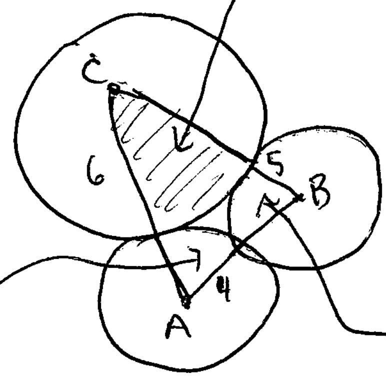
$$\boxed{\text{Area of Region} = \text{Area of } \Delta - \text{Area bounded by three arcs.}}$$

Side computation:
$$s = \frac{1}{2}(9+10+11) = 15.$$

$$\binom{\text{Area of}}{\Delta} = \sqrt{15(15-9)(15-10)(15-11)}$$
$$= \sqrt{15(6)(5)(4)} = 3\cdot 2\cdot 5\sqrt{3} = 30\sqrt{3}.$$

$\cdot \ \frac{\theta}{2} r^2$

$$\frac{\cos^{-1}(7/11)}{2} \cdot 6^2 \approx 15.85$$



$$\frac{\cos^{-1}(17/33)}{2} \cdot 5^2 \approx 12.87$$

$$\frac{\cos^{-1}(1/3)}{2} \cdot (4)^2$$

$$\approx 9.8472$$

$$\therefore \begin{pmatrix} \text{Area Bounded by} \\ \text{Arcs} \end{pmatrix} \approx 9.8472 + 12.87 + 15.85$$

$$= 38.5672$$

finally, from (☆)

$$\begin{pmatrix} \text{Area of} \\ \text{Region} \end{pmatrix} \approx 30\sqrt{3} - 38.5672$$

$$\approx 51.9615 - 38.5672$$

$$= 13.3943. \ //$$

7.1: 2

rewrite expression in terms of sine & cosine, & simplify

$$(\cos t)(\csc t) = (\cos t)\cdot\left(\frac{1}{\cos t}\right)$$

$$= 1.$$

7.1: 38

prove

$$\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = 1.$$

pf.

$$\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = \frac{\cos x}{1/\cos(x)} + \frac{\sin x}{1/\sin(x)}$$

$$= (\cos x)^2 + (\sin x)^2$$

$$= 1. //$$

7.2: 24

prove

$$\sin\left(x - \frac{\pi}{2}\right) = -\cos(x).$$

pf. Use the addition formula for sines.

$$\sin(\alpha+\beta) = \cos\alpha \sin\beta + \cos\beta \sin\alpha.$$

$$\sin\left(x - \frac{\pi}{2}\right) = \cos(x)\sin\left(\frac{-\pi}{2}\right) + \cos\left(\frac{-\pi}{2}\right)\sin(x)$$

$$= \cos(x)(-1) + (0)\cdot\sin(x) = -\cos(x). //$$

7.2: 32

prove

$$\cos(x-y) + \cos(x+y) = 2\cos(x)\cos(y)$$

cosine is even
sine is odd

pf:

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y).$$

$$\cos(x-y) = \cos(x)\cos(-y) - \sin(x)\sin(-y)$$

$$= \cos(x)(\cos(y)) - \sin(x)(-\sin(y))$$

$$= \cos(x)\cos(y) + \sin(x)\sin(y).$$

$\therefore$ $\cos(x+y) + \cos(x-y)$

$$= [\cos(x)\cos(y) - \sin(x)\sin(y)]$$

$$+ [\cos(x)\cos(y) + \sin(x)\sin(y)]$$

$$= 2\cos(x)\cos(y). \;//$$