HOMEWORK 4

8.4: 18, 24, 34
8.5: 6, 26

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18. $\vec{u} = (-2,5)$, $\vec{v} = (2,-8)$

$2\vec{u} = 2(-2,5) = (-4,10)$.

$-3\vec{v} = -3(2,-8) = (-6,24)$.

$$\begin{aligned}\vec{u}+\vec{v} &= (-2,5) + (2,-8) \\ &= (-2+2, 5-8) \\ &= (0,-3).\end{aligned}$$

$$\begin{aligned}3\vec{u} - 4\vec{v} &= 3(-2,5) - 4(2,-8) \\ &= (-6,15) + (-8,32) \\ &= (-14,47).\end{aligned}$$

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24. $\vec{u} = -2\hat{i} + 3\hat{j}$, $\vec{v} = \cancel{3\hat{i} - 2\hat{j}}$
$= (-2,3)$ $\quad = \cancel{(3,-2)}$.
$= \hat{i} - 2\hat{j} = (1,-2)$

$$\begin{aligned}\|\vec{u}\| &= \sqrt{(-2)^2 + 3^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13}.\end{aligned}$$

$$\|\vec{v}\| = \cancel{\sqrt{3^2+2^2}}$$
$$= \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}.$$

$$\|2\vec{u}\| = 2\|\vec{u}\|$$
$$= 2\sqrt{13}.$$

$$\|\tfrac{1}{2}\vec{v}\| = \tfrac{1}{2}\|\vec{v}\|$$
$$= \tfrac{1}{2}\sqrt{5}.$$

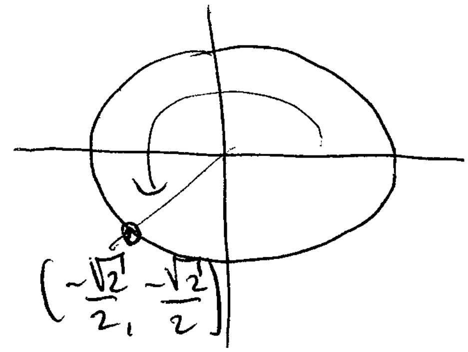
$$\|\vec{u}+\vec{v}\| = \|(-2,3)+(1,-2)\|$$
$$= \|(-1,1)\|$$
$$= \sqrt{1^2+1^2}$$
$$= \sqrt{2}.$$

$$\|\vec{u}-\vec{v}\| = \|(-2,3)-(1,-2)\|$$
$$= \|(-3,5)\|$$
$$= \sqrt{9+25}$$
$$= \sqrt{34}$$

, $34 = 2 \cdot 17$ so nothing comes out.

$\|\vec{u}\| - \|\vec{v}\| = \sqrt{13} - \sqrt{5}$, this doesn't simplify.

34. $\vec{v} = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$



$\|\vec{v}\| = 1.$ ← MAGNITUDE

$\theta = \frac{5\pi}{4}$

$= 5 \cdot 45° = 225°.$ ← DIRECTION.

8.5: 6

$\vec{u} = 2\hat{i} + \hat{j} = (2,1)$

$\vec{v} = 3\hat{i} + -2\hat{j} = (3,-2)$

(a) $\vec{u} \cdot \vec{v} = (2,1) \cdot (3,-2)$

$= 2 \cdot 3 + 1(-2)$

$= 6 - 2$

$= 4.$

(b) ANGLE VERSION OF DOT PRODUCT:

$$\boxed{\vec{u} \cdot \vec{v} = \|\vec{u}\|\|\vec{v}\| \cos\theta.}$$

$\|\vec{u}\| = \sqrt{2^2 + 1} = \sqrt{5}.$

$\|\vec{v}\| = \sqrt{9 + 4} = \sqrt{13}.$

$$\frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \cos\theta$$

$$\cos(\theta) = \frac{(4)}{(\sqrt{5})(\sqrt{13})} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{\sqrt{5}\sqrt{13}}\right).$$

26.

(a) Calculate $\text{proj}_{\vec{v}}(\vec{u})$.

NORMALIZE $\vec{v}$ & TAKE DOT PRODUCT OF $\vec{u}$ WITH $\vec{n}$ (the normalized vector) to get THE ~~VECTOR~~ PART OF $\vec{u}$ IN THE DIRECTION OF $\vec{v}$

$$\text{proj}_{\vec{v}}(\vec{u}) = \left(\vec{u} \cdot (\vec{n})\right) \vec{n}$$

normal scalar multiple.

where

$$\vec{n} = \frac{\vec{v}}{\|\vec{v}\|} \Big\} \text{ makes } \vec{n} \text{ have } \|\vec{v}\| = 1.$$

$$\therefore \text{proj}_{\vec{v}}(\vec{u}) = \left[\vec{u} \cdot \left(\frac{\vec{v}}{\|\vec{v}\|}\right)\right]\left(\frac{\vec{v}}{\|\vec{v}\|}\right)$$

$$= \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2}\vec{v} \quad \text{(formula from book)}$$

$\vec{u} = (11, 3)$
$\vec{v} = (-3, 2)$.

$$\|\vec{v}\|^2 = 3^2 + 2^2$$
$$= 9 + 4$$
$$= 13.$$

$$\vec{u} \cdot \vec{v} = (11, 3) \cdot (-3, 2)$$
$$= -33 \cancel{*} + 6$$
$$= \cancel{-27}. -39$$

$$\therefore \operatorname{proj}_{\vec{v}}(\vec{u}) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v}$$
$$= \frac{(\cancel{-27} \, 39)}{(13)} (-3, 2)$$
$$= \cancel{\left(\frac{81}{13}, \frac{-54}{13}\right)}. \; (9, 6)$$

(b) Resolve $\vec{u}$ into $\vec{u}_1$ & $\vec{u}_2$ where $\vec{u}_1$ parallel to $\vec{v}$ and $\vec{u}_2$ perp to $\vec{v}$,

write $\boxed{\vec{u} = \vec{u}_1 + \vec{u}_2}$

$$\vec{u}_1 = \operatorname{proj}_{\vec{v}}(\vec{u})$$
$$= \cancel{\left(\frac{81}{13}, \frac{-54}{13}\right)}. \; (9, 6$$

$$\vec{u}_2 = \vec{u} - \vec{u}_1$$

over

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$$(9, 6)$$

$$= (11, 3) - \left(\frac{81}{13}, \frac{-54}{13}\right)$$

$$= \left(11 - \frac{81}{13}, 3 + \frac{54}{13}\right)$$

$$= \left(\frac{141 - 81}{13}, \frac{39 + 54}{13}\right)$$

$$= \left(\frac{60}{13}, \frac{93}{13}\right).$$

$$= (2, -3)$$