Practice Test 1

Math 150 —Dupuy

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No calculators. You must work on the test by yourself.

1. Simplify the rational expression
$$\frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}}$$

2. Verify that the equation
$$x^2+y^2-10x+4y+20=0$$
defines a circle. At what point is the circle centered? What is the radius of the circle?

3. Find an expression for the surface area of a cube as a function of its Volume.

4. Graph $y=2^{x-2}+4$.

5. Let $f(x)=x^2+2$, $g(x)=e^x$ and $h(x)=1/x$,
   (a) Compute $(f\circ g\circ h)(x)$.
   (b) Compute $(g\circ f\circ g)(x)$.

6. Let $f(x)=x^2-4x+8$.
   (a) Express $f(x)$ in standard form $f(x)=c(x-x_0)^2+b$.
   (b) Where does f achieve its maximum or minimum?
   (c) Graph $f(x)$.

7. Solve the following equations for x:
   (a) $e^{x^2+2}=14$.
   (b) $\ln(x+1)+\ln(x)=\frac{1}{2}$.

8. Radioactive decay is modeled by the equation $m(t)=m_0e^{-kt}$, where t is in days and $m(t)$ is in grams.
   (a) If the half-life of the element is $t=\ln 2$ days find the constant k. (half life is the time it takes for the mass to be reduced by half.)
   (b) If in three days the mass remaining is e^{-3} kg find how much mass there was at time $t=0$.

9. Expand the expression
$$\ln\left(\frac{\sqrt{x+1}(x-1)}{x^6}\right).$$

10. State the Fundamental Theorem of Algebra *correctly.*

11. Given the information about the polynomial, find its explicit formula:
   (a) $P(x)$ is a polynomial of degree 3 with $r_1=1$ as a root of multiplicity 2, and $r_2=2$ as a root of multiplicity 1. In addition $P(x)$ satisfied $P(0)=1$.
   (b) $f(x)$ is a polynomial of degree 3 with *integer coefficients.* It has $r_1=3$ and $r_2=-2i$ as roots.

12. Factor the polynomials completely:

    (a) $x^3 - 27$.

    (b) $x^4 + 2x^2 - 15$.

13. Consider the polynomial $f(x) = x^5 - x^4 - x^3 - x^3 - x - 2$.

    (a) Verify that $r = 2$ is a root of $f(x)$.

    (b) Using the division algorithm, factor of the term of $f(x)$ corresponding to the root $r = 2$.

14. Consider the polynomial $f(x) = x^4 - 3x^3 + 3x^2 - 3x + 2$.

    (a) According to the rational roots theorem what are the possible rational roots of $f(x)$?

    (b) I'll tell you that all of the rational roots of this polynomial are positive numbers, factor it completely. (This part included testing for rational roots, finding a factor of degree 2)

15. Write the following complex numbers in the form $a + ib$.

    (a) $\frac{1+i}{1-i}$.

    (b) $\frac{1}{2+i} - \frac{2}{2-i}$.

16. (Extra Credit) Who was the mathematician that related the number $e = 2.71828182\ldots$ to the compounding interest problem?