Practice Test 3

Dupuy — Math 150 — Summer 2008

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1. State the definition of a derivative of function $f(x)$ at a point x_0.
2. Compute the derivative of the function $f(x) = x^3$ *via a difference quotient*.
3. Complete the following rules for derivatives. Below, $f(x)$ and $g(x)$ are functions, $c \in \mathbb{R}$ and $n \in \mathbb{N}$.
   (a) $\frac{d}{dx}[c] =$
   (b) $\frac{d}{dx}[f(x) + g(x)] =$
   (c) $\frac{d}{dx}[x^n] =$
4. (a) State the product rule.
   (b) Prove the product rule by using the definition of the derivative (as in the homework).
5. Compute the derivatives of the following polynomials
   (a) $-2x^3 + x + 1$
   (b) $x^{500} + 1500$
   (c) $\alpha x^4 + 5\beta x^2 - 2\gamma x$, where α, β and γ are real numbers.
6. Find all the solutions of the system

$$\begin{cases} y = x^2 - 2x + 2 \\ y = x^2 - 2x \end{cases} .$$

7. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.
   (a) Suppose A is invertible. State the formula for A^{-1}.
   (b) Compute the inverse of $B = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.
8. Let $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}$. Compute AB.
9. Put the following 3×3 system of into matrix form and solve by Gaussian Elimination

$$\begin{cases} x - z = 2 \\ -2x + y + 2z = 0 \\ -6x + 3y + 7z = 1 \end{cases} .$$

10. (Extra Credit) Define what an eigenvector of a Matrix is. Define what an eigenvalue of a matrix is. Compute the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}. \qquad (1)$$