Exercises On Sets

Math 150 —Dupuy

June 23, 2008

1. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8, 10\}$. Compute the following sets.

   (a) $A \cap B$.
   (b) $A \cup B$.
   (c) $A \setminus B$.
   (d) $B \setminus A$.
   (e) $A \times B$.
   (f) $\{a \in A \; : \; a$ is even $\}$.
   (g) $\{x \in A \; : \; x + 1 \in B \}$.
   (h) $\{x \in A \; : \; x + 1 \in B$ or $x + 5 \in B\}$.

2. Let $A = \mathbb{N}$ and let $B = \{n \in \mathbb{N} \; : \; n$ is even $\}$. Find the following sets

   (a) $A \cap B$.
   (b) $A \cup B$.
   (c) $A \setminus B$.
   (d) $B \setminus A$.

3. Order the following sets via inclusion. Example: the sets $\{1, 2, 3\}, \{2\}, \{2, 1\}$ satisfy

$$\{2\} \subset \{2, 1\} \subset \{1, 2, 3\}.$$

   (a) $\{1, 2, 4\}, \mathbb{N}, \{1, 2\}, \emptyset$
   (b) $\mathbb{R}, \mathbb{C}, \mathbb{Z}, \mathbb{Q}, \mathbb{N}$.

4. State the precise definition of a function.

5. If the domain for a function is $A = \{1, 2, 3, 4, 5\}$ and the range is $B = \}1, 2, 3, 4, 5\}$ determine if the following relations are functions. If they aren't, explain what part of the function definition they violate.

   (a) $S = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 4)\}$.
   (b) $T = \{(1, 2), (1, 3), (2, 4), (3, 5), (5, 1)\}$.
   (c) $U = \{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1)\}$.
   (d) $V = \{(1, 1), (2, 1), (3, 1), (4, 1)\}$

6. Write the following sets using set notation. Example $(1, 6) = \{x \in \mathbb{R} \; : \; 1 < x < 6\}$.

   (a) $[0, 2)$
   (b) $(-\infty, 2)$
   (c) $[-1, 1]$

7. Find y as a function of x. In each case, specify how many functions the relations below determine. Also, when specifying a function $y = f(x)$ state a range and a domain for f in set notation, assume that f is a real valued function. Example: If $f(x) = 1/x$ then $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$.

(a) $x^2 + 2y = 4$

(b) $x = y^2$

(c) $x^2 y^2 + y = 1$

8. Find the image of the following functions. Remember: the image of a functions $f : A \to B$ is defined by $Im(f) = \{f(a) \in B \ : \ a \in A\}$.

   (a) $f : \mathbb{N} \to \mathbb{N}$ defined by $f(n) = 2n$.

   (b) $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined by $f(x, y) = x^2 + y^2$.

9. The Dirichlet function $D : \mathbb{R} \to \{0, 1\}$ is defined by

$$f(x) = \begin{cases} 0, & x \in \mathbb{Q} \\ 1, & x \text{ not in } Q \end{cases}$$

what is the image of f?