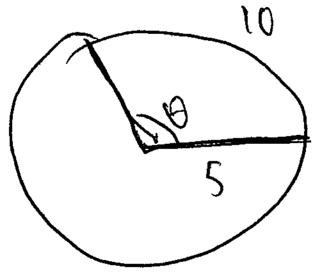
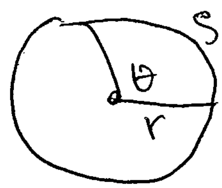
HOMEWORK #2

6.1: 50
6.2: 6, 23
6.3: 38, 58

6.1:50: FIND THE ANGLE ~~AREA~~ IN THE FIGURE



SOLN: In general



$\boxed{\theta r = s}$

In our case, $r=5$ & $s=10$. This means

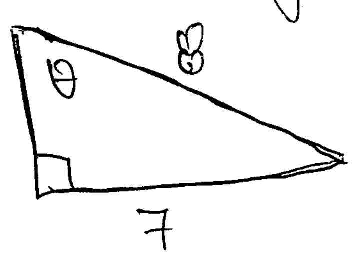
$$\theta \cdot 5 = 10$$

$\Rightarrow \boxed{\theta = 2}$. (Note the angle is given in radians)

//

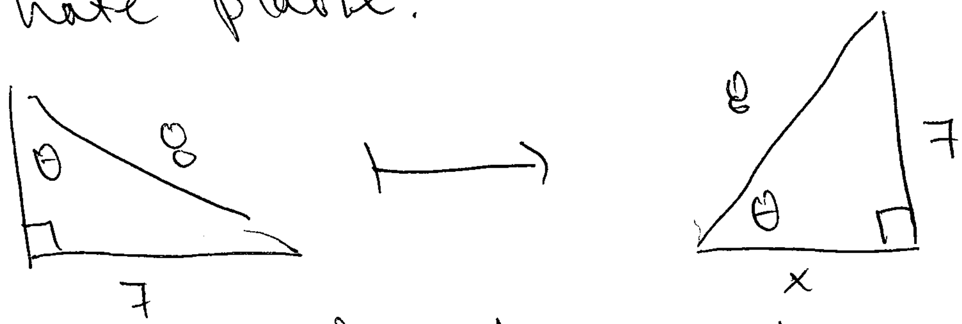
6.2: 6:

Find the values of the six trigonometric functions for the angle θ where



THERE ARE MANY WAYS TO DO THIS!

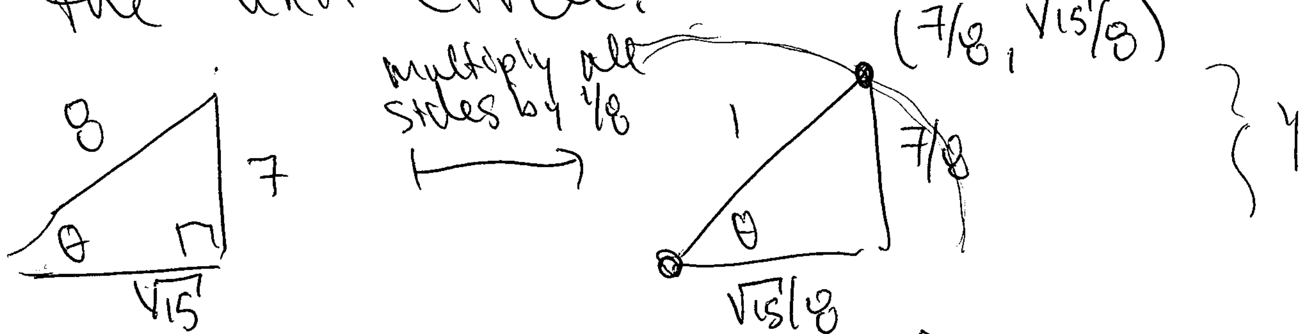
ANS: First rotate the triangle so that it looks like we expect it to in the coordinate plane:



- We can solve for the missing side x by using the pythagorean theorem.

$$8^2 = 7^2 + x^2$$
$$\Rightarrow 64 - 49 = x^2$$
$$\Rightarrow \sqrt{15} = x.$$

- Scale our triangle so that it fits on the unit circle.



- use the defns of the six trig fns.

DEFNS

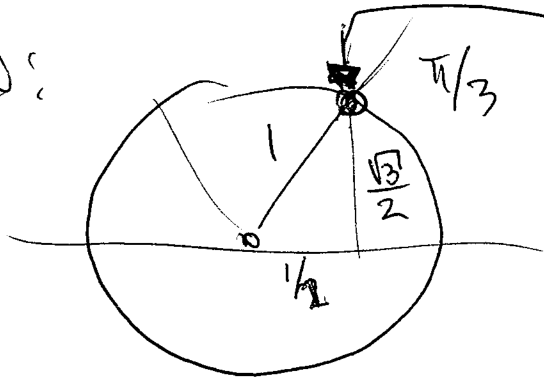
$\sin(\theta) = y$	$\csc(\theta) = \frac{1}{y}$
$\cos(\theta) = x$	
$\tan(\theta) = y/x$	
$\cot(\theta) = x/y$	
$\sec(\theta) = \cancel{1/\cos(\theta)} = 1/x$	

$\Rightarrow$

$$\boxed{\begin{aligned} \sin(\theta) &= 7/8 \\ \cos(\theta) &= \sqrt{15}/8 \\ \tan(\theta) &= 7/\sqrt{15} \\ \cot(\theta) &= \sqrt{15}/7 \\ \sec(\theta) &= 8/\sqrt{15} \\ \csc(\theta) &= 8/7 \end{aligned}}$$

6.2:23 Compute the value of $\sin(\pi/3) + \cos(\pi/3)$.

SOLN:



$(\cos(\pi/3), \sin(\pi/3)) = (1/2, \sqrt{3}/2)$

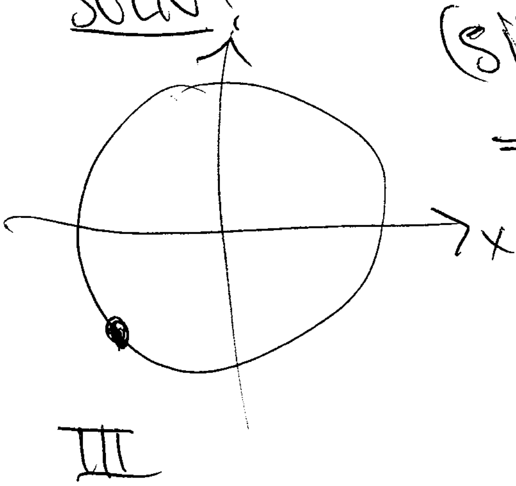
so $\begin{cases} \cos(\pi/3) = 1/2 \\ \sin(\pi/3) = \sqrt{3}/2 \end{cases}$

($\frac{\pi}{3} = 60°$ so we have a $30°, 60°, 90°$ triangle.)

$$\sin(\pi/3) + \cos(\pi/3) = \sqrt{3}/2 + 1/2 = \boxed{\frac{1+\sqrt{3}}{2}}. //$$

6.3:38 Write $\cot(\theta)$ in terms of $\sin(\theta)$ given that the point corresponding to θ in in quadrant 3.

SOLN:



III

$$(\sin\theta)^2 + (\cos\theta)^2 = 1$$

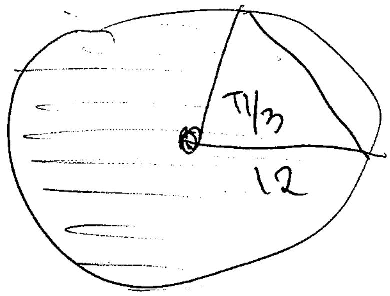
$$\Rightarrow \cos\theta = \pm\sqrt{1 - \sin(\theta)^2}$$

given that we are in quadrant III $\cos\theta$ is <u>negative</u> so

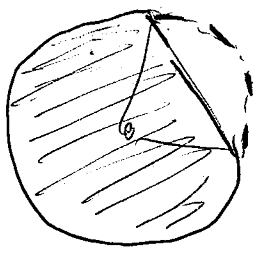
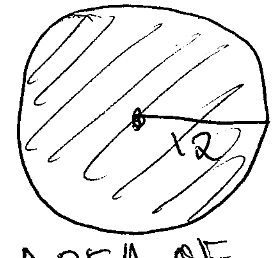
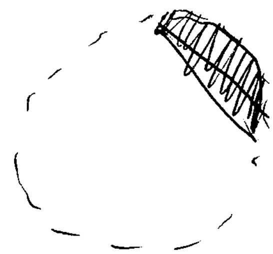
$$\cos\theta = -\sqrt{1 - \sin(\theta)^2}.$$

$$\therefore \cot(\theta) = \frac{\cos\theta}{\sin\theta} = \frac{-\sqrt{1-(\sin\theta)^2}}{\sin\theta}. //$$

6.3.58: Find the area of the shaded region.



SOLN:

 =  − 

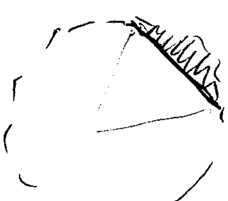
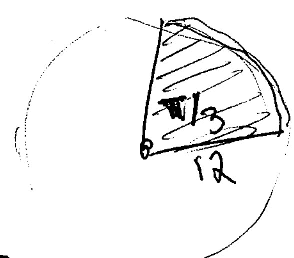
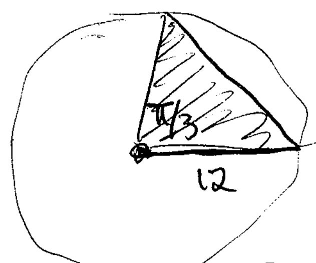
OUR AREA

AREA OF CIRCLE

LITTLE PIECE AREA

$\pi(12)^2 = \pi 144.$

But,

 =  − 

LITTLE PIECE AREA

WHOLE SECTOR AREA

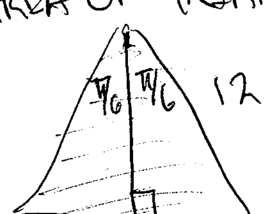
AREA OF TRIANGLE

$$\underbrace{\frac{\theta}{2} r^2}_{\text{formula for area of sector}} = \frac{(\pi/3)}{2}(12)^2$$

$36\sqrt{3}$

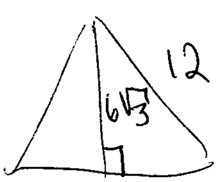
WORK DOWN HERE

AREA OF TRIANGLE

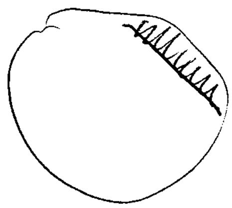


$12\cos(\pi/6)$
$= 12(\sqrt{3}/2)$
$= 6\sqrt{3}.$

$12\sin(\pi/6)$
$= 12\cdot(1/2) = 6.$

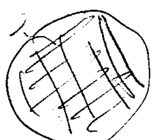
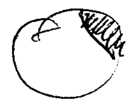


$$\text{AREA OF TRIANG} = \frac{1}{2} b h = \frac{1}{2}(6+6)(6\sqrt{3}) = 36\sqrt{3}.$$

So,  $= \frac{(\pi/3)}{2}\cdot(12)^2 - 36\sqrt{3}$

$= \cancel{}\ \frac{\pi}{6}\cdot 144 - 36\sqrt{3}$,

$\Downarrow$

OUR AREA $= \pi\, 144 - \left(\frac{\pi}{6}\cdot 144 - 36\sqrt{3}\right)$

$= \frac{5\pi}{6}\cdot 144 + 36\sqrt{3}$

$= \pi\cdot 120 + 36\sqrt{3}$. //