Practice Problems for 123 Final

Summer 2009

July 28, 2009

Here are some problems that you can practice. The idea is to remind you of some of the sections we have covered. Some of them require answers in sentence form. As a reminder, all of the types of problems we have done on previous tests and homeworks are fair game for the final.

1. Find all the solutions of $z^4 = 16i$.
2. Express $\sec(\tan^{-1}(x))$ as an algebraic expression involving x.
3. Write $\frac{2+3i}{1+i}$ in cartesian form $x + iy$.
4. Express $\tan\theta$ as a function of $\csc\theta$.
5. Using $(\cos\theta)^2 + (\sin\theta)^2 = 1$ prove that
$$(\cot\theta)^2 + 1 = (\csc\theta)^2.$$
6. How did we define the radian angle?
7. What is the formula for the area of a sector with angle θ and radius r?
8. If a given sector as area 1 and radius 3, what is the angle for that sector?
9. How did we define $\cos\theta$ and $\sin\theta$ at the beginning of class.
10. Why does $(\cos\theta)^2 + (\sin\theta)^2 = 1$?
11. Convert the points given in cartesian form to polar form:
    (a) $(1, 1)$.
    (b) $(2, 3)$.
    (c) $(3, -3\sqrt{3}/2)$.
12. Convert the polar equation to a cartesian one:
$$r = \frac{b}{\sin\theta - m\cos\theta}.$$
13. Convert the cartesian equation to a polar one:
$$x^2 = y - y^2.$$

14. Using the difference of angles formula derive

$$\cos(2\theta) = (\cos\theta)^2 - (\sin\theta)^2.$$

15. Graph $5\sin(2t-1)$. Find and label the period, amplitude, phase shift (be careful!), and angular frequency.

16. Let $\vec{v} = (2, -1)$ and $\vec{w} = (3, 3)$. Compute the following

    - $2\vec{v} + \vec{w}$.
    - $\|\vec{v}\|$.
    - $\vec{v} \cdot \vec{w}$

17. Explain how the law of cosines (in triangle form) gives the pythagorean theorem for right triangles.

18. Suppose $\vec{v}$ is a vector with $\|\vec{v}\| = 2$ and $\vec{w}$ is a vector with $\|\vec{w}\| = 1$. Further, suppose that the angle between $\vec{v}$ and $\vec{w}$ is $\pi/4$. Compute $\vec{v} \cdot \vec{w}$.

19. How can you test using the dot product if two vectors are perpendicular?

20. Are the vectors $\vec{v} = (-2, 1)$ and $\vec{v} = (4, 8)$ perpendicular?