HOMEWORK & SOLNS

1. Find cube roots of $-27i$.

$$-27i = 27e^{i\pi}e^{i\pi/2}\cdot 1$$
$$= 27e^{i3\pi/2}e^{i2k\pi} \Rightarrow$$
$$= 27e^{i\left(\frac{3\pi}{2}+2k\pi\right)}$$

$$(-27i)^{1/3} = \left(27e^{i\left(\frac{3\pi}{2}+2k\pi\right)}\right)^{1/3}$$
$$= (27)^{1/3}e^{i\left(\frac{3\pi}{6}+\frac{2k\pi}{3}\right)}$$
$$= 27^{1/3}e^{i\pi/2}e^{i\frac{2k\pi}{3}}$$
$$= 3ie^{i\frac{2k\pi}{3}}$$

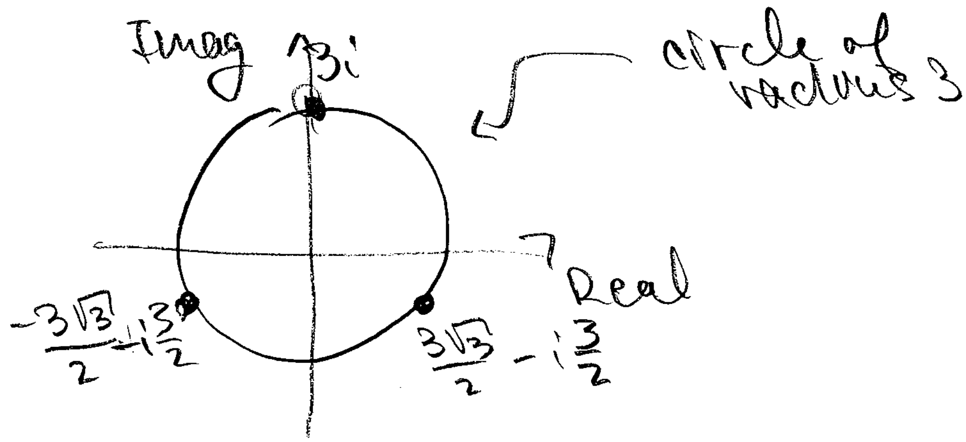
$\underline{k=0}$: $w_0 = 3i$

$\underline{k=1}$: $w_1 = 3i\left(e^{i\frac{2\pi}{3}}\right) = 3i\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$$= 3i\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$
$$= -\frac{3\sqrt{3}}{2} - i\frac{3}{2}.$$

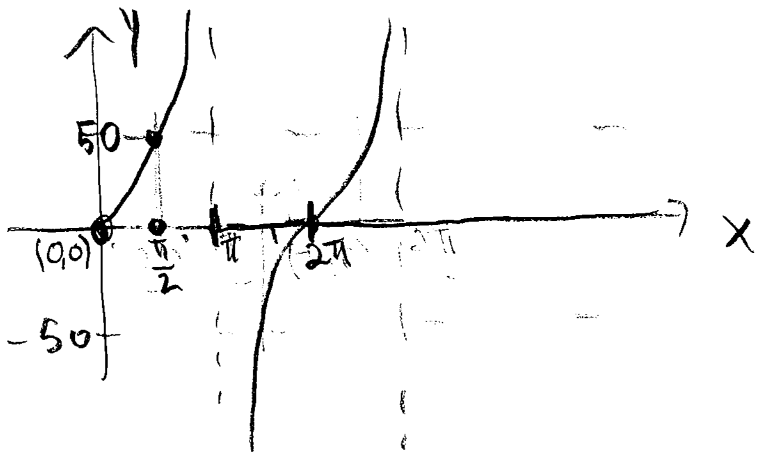
$\underline{k=2}$: $w_2 = 3i\left(e^{i4\pi/3}\right) = 3i\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

$$= \frac{3\sqrt{3}}{2} - i\frac{3}{2}.$$



2. $\boxed{\text{Graph } y = 50\tan\left(\frac{x}{2}+\pi\right).}$

$$50\tan\left(\frac{x}{2}+\pi\right) = 50\tan\left(\frac{x}{2}+\frac{2\pi}{2}\right)$$
$$= 50\tan\left(\frac{1}{2}(x+2\pi)\right)$$



- period $= 2\pi$
- the shift of 2π makes no difference

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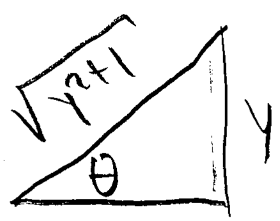
3. $\boxed{\sin(\tan^{-1}(y)) = ?}$

Let $\theta = \tan^{-1}(y)$. $\Rightarrow$ $y = \tan(\theta)$

Here is a triangle where $\tan\theta = y$:



Using this triangle we see that $\sin\theta = \frac{y}{\sqrt{1+y^2}}$

$$\therefore \sin(\tan^{-1}(y)) = \sin\theta$$
$$= \frac{y}{\sqrt{1+y^2}}. //$$

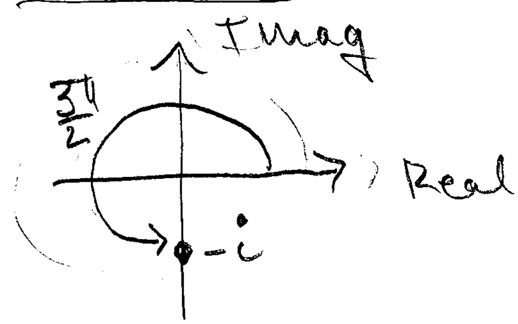
4. Find z_1/z_2 in polar & cartesian form:

$$\boxed{\begin{cases} z_1 = 2\sqrt{3} - 2i \\ z_2 = 1 + \sqrt{3}i \end{cases}}$$

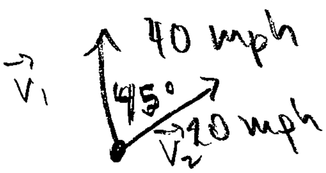
$$\frac{z_1}{z_2} = \frac{2\sqrt{3} - 2i}{1+\sqrt{3}i} = \frac{2(\sqrt{3} - i)}{1+\sqrt{3}i} = \frac{-2i(1+\sqrt{3}i)}{1+\sqrt{3}i} = \boxed{-2i}.$$

Cartesian Form,

Since $-i = e^{i\frac{3\pi}{2}}$ see graph



the polar form is $\boxed{2e^{i3\pi/2}}$.

5. 

$$\vec{x}_1 = \vec{v}_1 t$$
$$\vec{x}_2 = \vec{v}_2 t$$

$\longleftarrow$ positions at time t,

$$\|\vec{x}_1 - \vec{x}_2\|^2 = \|\vec{x}_1\|^2 + \|\vec{x}_2\|^2 - 2\|x_1\|\,\|x_2\|\cos(\theta)$$

difference in positions

plug in various t's to get answers.

$$\Rightarrow d^2 = \|\vec{v}_1 t\|^2 + \|\vec{v}_2 t\|^2 - 2\|\vec{v}_1 t\|\,\|\vec{v}_2 t\|\frac{\sqrt{2}}{2}$$

$$= t^2 40^2 + t^2 20^2 - 2(20t)(40t)\frac{\sqrt{2}}{2}$$

$$= t^2(1600 + 400 - \sqrt{2}\,800) = t^2(2000 - \sqrt{2}\,800)$$